Applied Mathematics and Computation 289 (2016) 132–148

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Deblurring Poisson noisy images by total variation with overlapping group sparsity☆

Xiao-Guang Lv[a,b,*], Le Jiang[a], Jun Liu[c]

[a] School of Science, Huaihai Institute of Technology, Lianyungang, Jiangsu 222005, China
[b] School of Mathematical Sciences, Nanjing Normal University, Nanjing, Jiangsu 210097, China
[c] School of Mathematics and Statistics, Northeast Normal University, Changchun, Jilin 130024, China

ARTICLE INFO

Keywords:
Deblurring
Total variation
Overlapping group sparsity
Alternating direction method of multipliers
Poisson noise

ABSTRACT

Deblurring Poisson noisy images has recently been subject of an increasingly amount of works in various applications such as astronomical imaging, fluorescent confocal microscopy imaging, single particle emission computed tomography (SPECT) and positron emission tomography (PET). Many works promote the introduction of an explicit prior on the solution to regularize the ill-posed inverse problem for improving the quality of the images. In this paper, we consider using the total variation with overlapping group sparsity as a prior information. The proposed method can avoid staircase effect and preserve edges in the restored images. After converting the proposed model to a constrained problem by variable splitting, we solve the corresponding problem with the alternating direction method of multipliers (ADMM). Numerical examples for deblurring Poisson noisy images are given to show that the proposed method outperforms some existing methods in terms of the signal-to-noise ratio, relative error and structural similarity index.

1. Introduction

Deblurring images corrupted by Poisson noise is an important task in various applications such as astronomical imaging, fluorescent confocal microscopy imaging, single particle emission computed tomography (SPECT) and positron emission tomography (PET) [1–7]. In these applications, the image formation can be modeled as a linear process including deterministic and statistical aspects. More precisely, we denote by $g \in \mathbb{R}^{n\times n}$ the recorded image, each measured value g_{ij} is a realization of a Poisson random variable with expected value $(Hu+b)_{i,j}$, where $u \in \mathbb{R}^{n\times n}$ is the original image, $b \geq 0$ is a nonnegative constant background and $H : \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$ is a blurring operator related with the spread point function (PSF) and the boundary conditions. Generally, the exact structure of H depends on the imposed boundary conditions. In practice, the boundary conditions such as periodic boundary conditions, zero boundary conditions, reflexive boundary conditions and antireflexive boundary conditions are often chosen for algebraic and computational convenience [8,9].

The goal of Poisson noisy image deblurring is to reconstruct an approximation of the original image from the observed image g. A natural way to attack this problem would be to adopt a maximum a posteriori (MAP) bayesian framework with

☆ This research is supported by NSFC (61401172), 973 Program (2011CB707104), Nature Science Foundation of Jiangsu Province (BK20131209), Nature Science Foundation of HHIT (Z2015004), and Postdoctoral Research Funds (2013M540454, 1301064B).

* Corresponding author at: School of Science, Huaihai Institute of Technology, Lianyungang, Jiangsu 222005, China. Tel.: +8615961307352.
E-mail address: xiaoguanglv@126.com, xiaoguanglv@hhit.edu.cn (X.-G. Lv).

http://dx.doi.org/10.1016/j.amc.2016.03.029

an appropriate likelihood function—the distribution of the observed data g given an original image u—reflecting the Poisson statistics of the noise. It is known that the probability distribution of g can be written as

$$\mathcal{L}(u) = P(g|u) = \prod_{i,j=1}^{n} \frac{[(Hu+b)_{i,j}]^{g_{i,j}}}{g_{i,j}!} e^{-(Hu+b)_{i,j}}. \tag{1}$$

By taking the negative logarithm of the likelihood function and adding a suitable constant, we obtain the Kullback–Leibler divergence of $Hu + b$ from g [10]:

$$\mathcal{T}_0(u) = -\log[L(u)] = \sum_{i,j=1}^{n} (Hu+b)_{i,j} - g_{i,j} + g_{i,j}\log\frac{g_{i,j}}{(Hu+b)_{i,j}}. \tag{2}$$

After dropping some terms independent of u in $\mathcal{T}_0(u)$, we obtain

$$\mathcal{J}_0(u) = \sum_{i,j=1}^{n} (Hu+b)_{i,j} - g_{i,j}\log(Hu+b)_{i,j}. \tag{3}$$

Therefore, the maximum likelihood estimator of the original image is the minimizer with respect to u of the negative-log Poisson likelihood functional $\mathcal{J}_0(u)$. We know that the function $\mathcal{J}_0(u)$ is strictly convex if the blurring operator H is nonsingular and in such a case the minimizer is unique. It is obvious that, if the equation $Hu + b = g$, has a nonnegative solution, then this is also a minimizer of $\mathcal{J}_0(u)$. However, H from the practical image deblurring problem has many singular values of different orders of magnitude close to the origin. Because of the extreme ill-conditioning of H, such a solution, in general, does not exist and, as a consequence, a minimizer of $\mathcal{J}_0(u)$ may not provide a sensible solution of the image deblurring problem [11].

The problem of deblurring Poisson noisy images has received considerable attention in recent years. In [12,13], Richardson and Lucy presented a deconvolution method for Poisson noisy image deblurring, named as the R-L algorithm. It consists on the iterative minimization of the non-quadratic log-likelihood function $\mathcal{J}_0(u)$ with multiplicative corrections. The main shortcoming of this method is that after a few iterations, the algorithm yields highly noisy estimates, in particular when the signal-to-noise ratio (SNR) is low [14]. Alternatively, many authors promote the introduction of an explicit prior on the solution to regularize the ill-posed inverse problem and thus minimize a penalized likelihood [15]. Usually, regularization methods formulate the image deblurring problem as a minimization problem of the form

$$\mathcal{J}(u) = \mathcal{J}_0(u) + \alpha\mathcal{J}_R(u), \tag{4}$$

where the regularization function $\mathcal{J}_R(u)$ incorporates prior information about the object to be recovered and $\alpha > 0$ is the regularization parameter that controls the balance between the fidelity term $\mathcal{J}_0(u)$ and the regularization term $\mathcal{J}_R(u)$. In the Bayesian approach, $\mathcal{J}(u)$ can be obtained by maximizing the posterior density $P(u|g) = P(g|u)P(u)/P(g)$ with respect to u. The maximizer of $P(u|g)$ is called the MAP estimate. In fact, $\mathcal{J}_R(u) = -\log(P(u))$ where $P(u)$ is a prior probability density from which the unknown u assumed to arise.

In general, the choice of the regularization function $\mathcal{J}_R(u)$ is related to the features of the images to be restored. How to choose a good functional $\mathcal{J}_R(u)$ is an active area of research in the imaging science. Probably one of the most popular regularization methods is Tikhonov regularization [16], which corresponds to quadratic functionals of the form $\mathcal{J}_R(u) = \|Lu\|_2^2$, where L is usually chosen to be the identity operator or differentiation operator. The Tikhonov regularization method is suitable when the image to be restored is made by diffused objects, such as astronomical images of nebula or certain images from microscopy [17]. In [18], Landi and Piccolomini proposed a quasi-Newton projection method for deblurring of Poisson-corrupted images by solving a nonnegatively constrained minimization problem where the objective function is the sum of the Kullback–Leibler divergence, used to express fidelity to the data in the presence of Poisson noise, and a Tikhonov regularization term. In [19,20], the authors proposed an efficient hybrid gradient projection-reduced Newton method for the problem arising in astronomical imaging, by developing a cost functional which incorporates the statistics of the Poisson noise in the image data and the Tikhonov regularization to induce stability. In [21], the authors presented a computational study on scaling techniques in gradient projection-type methods for deblurring of Poisson-corrupted astronomical images by minimizing an objective function, which takes the Kullback–Leibler divergence as the fidelity term and the Tikhonov regularization function as the penalty term.

Although Tikhonov regularization has the advantage of simple calculations, it produces a smoothing effect on the restored images, especially when L is the differentiation matrix. To overcome this shortcoming, Rudin et al. [22] proposed a total variation (TV)-based regularization technique, which preserves the edge information in the restored image. In the case of TV regularization, the estimated solution for Poisson noisy image deblurring is obtained by solving the following problem:

$$\min_u \mathcal{J}(u) \doteq \mathcal{J}_0(u) + \alpha\|\nabla u\|_1. \tag{5}$$

The discrete gradient operator $\nabla u : \mathbb{R}^{n\times n} \to (\mathbb{R}^{n\times n}, \mathbb{R}^{n\times n})$ is defined by

$$(\nabla u)_{i,j} = ((\nabla_x u)_{i,j}, (\nabla_y u)_{i,j})$$