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Radiation effects from an isothermal vertical wavy cone with variable fluid properties

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ABSTRACT

Numerical solutions are presented for the natural convection flow along a vertical wavy cone situated in the thermally radiating fluid. The fluid flow and heat transfer characteristics are analyzed for the fluid having temperature dependent viscosity and thermal conductivity. After the primitive variable formulations, the transformed equations are integrated numerically through implicit finite difference method. Computational results are carried out for a range of physical parameters and interpreted in the form of skin friction coefficient, Nusselt number coefficient, streamlines and isotherms. The calculations show strong influence of thermal radiation parameter on the velocity and temperature fields. It is also reported that variable fluid properties sufficiently alter the important physical quantities and the quantitative analysis determines that it is likely to be more than 50%.

1. Introduction

The analysis of laminar natural convection flow along an irregular (wavy) surface is of fundamental importance because it is often used in various practical and engineering applications. For instance, heat transfer devices like flat-plate solar collectors, industrial heat exchangers and condensers in refrigerators possess non-uniform surfaces. The corresponding problem of heat transfer to an incompressible fluid from a non-uniform vertical surface was first considered by Yao [1] and Moulic and Yao [2]. The class of problems discussed by the authors Yao [1] and Moulic and Yao [2] were probably the most interesting one; both mathematically and physically. Keeping in view [1,2], several investigations have been done by taking practical situations into account. Specifically, authors in Refs. [3–8] considered the vertical and/or horizontal wavy surface under different circumstances and reported significant results. Other than this, effects of non-uniformities were also discussed for wavy cone. In this context, Pop and Na [9] investigated both the constant wall temperature and uniform heat flux distribution on natural convection flow along a vertical wavy cone. Moreover, Hossain et al. [10] reported the natural convection flow along a vertical wavy cone with variable fluid properties. Recently, Shermet et al. [11] presented a numerical investigation on the unsteady free convection heat transfer characteristics of a nanofluid, confined within a porous open wavy cavity, using the mathematical model proposed by Buongiorno. The wavy-walled cavity was considered in the study conducted by Sheremet and Miroshnichenko [12], in which the authors reported the effect of thermal radiation on unsteady natural convection flow.

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It can be further noted that some fluids have significant role in the theory of lubrication, for instance, they do not correspond to uniform nature of viscosity and thermal conductivity since the heat generated by the internal friction and the corresponding rise in temperature deliberately alter these properties. Many researchers exploit this idea by exposing appropriate temperature dependent relations of fluid viscosity and thermal conductivity, which ultimately improves the accuracy of the results up to a significant ratio. In addition, many problems in engineering occur at very high temperature and hence the knowledge of viscosity effects on the convective heat transfer becomes very important for the design of the pertinent equipment. For instance, the viscosity of water increases by about 240%, when the temperature decreases from 50 °C ($\mu = 0.000548\ \mathrm{kgm^{-1}s^{-1}}$) to 10 °C ($\mu = 0.00131\ \mathrm{kgm^{-1}s^{-1}}$). The temperature dependent properties of viscous fluid are of considerable importance in lubrication, tribology, instrumentation, food processing and viscometery. Several studies, for example see Refs. [13–20], have introduced temperature dependent properties and reported significant influence of these properties over the flow characteristics. In these articles, viscosity of the fluid has been considered to be inversely proportional to the linear function of temperature. There are very few forms of variable viscosity and thermal conductivity available in the literature, out of which following forms are considered in the present analysis:

$$\mu = \mu_\infty\left[1 + \frac{\varepsilon(T - T_\infty)}{(T_w - T_\infty)}\right], \quad \kappa = \kappa_\infty\left[1 + \frac{\gamma(T - T_\infty)}{(T_w - T_\infty)}\right] \tag{1}$$

The semi-empirical expressions given above are appropriate for liquids and introduced by Charraudeau [20]. Here, μ_∞ and κ_∞ are the viscosity and thermal conductivity of the ambient fluid, and ε and γ are, respectively, the viscosity variation parameter and thermal conductivity variation parameter.

In the present analysis, consideration has been given to the effects of radiative heat transfer on laminar natural convection flow moving along the vertical wavy cone. Such problems found their applications in many engineering processes, such as storage of radioactive materials, semi-conductor wafers, nuclear reactors, advanced power plants and processes involving high temperature. Many authors have reported thermal radiation effects by using Rosseland diffusion approximation (see Ref. [21]) for an optically dense medium. For instance, investigations on the natural convection flow as well as on the mixed convection flow of an optically dense gray viscous fluid past or along heated bodies of different geometries, such as, vertical and horizontal flat plate, cylinder, sphere, wavy surface and axisymmetric rotating and non-rotating bodies under different boundary conditions have been accomplished by the authors of [22–28]. Thus, present paper explains the behavior of natural convection flow along a vertical wavy cone with thermal radiation effects. It can be noted that cone shape geometries are commonly used in multiple industry applications to capture and remove debris prior to putting a pipeline into production service. The fluid properties, namely, fluid viscosity and thermal conductivity are assumed to take the form as proposed in (1). Taking Grashof number Gr to be very large, the boundary layer approximation is invoked, leading to a set of non-similar parabolic partial differential equations, whose solution is obtained through implicit finite difference method. Consideration has been given to the situation where the buoyancy forces assist the natural convection flow for various combinations of the radiation–conduction parameter or Plank constant, R_d, surface heating parameter, θ_w, viscosity variation parameter, ε, and the thermal conductivity variation parameter, γ.

2. Analysis

Consider the steady two-dimensional natural convection flow of a dense fluid moving along the vertical wavy cone. The boundary layer analysis outlined below allows the shape of the wavy surface, $\hat{\sigma}(\hat{x})$, to be arbitrary, but our detailed numerical work will assume that the surface exhibits sinusoidal deformations. Thus, the wavy surface of the cone is described by:

$$\hat{y}_w = \hat{\sigma}\left(\hat{x}\right) = \hat{a}\sin\left(\frac{2\pi\hat{x}}{L}\right) \tag{2}$$

where $\hat{a}$ and L are, respectively, the dimensional amplitude of the wavy cone and characteristic length associated with the uneven surface (also known as half of the wavelength of the uneven surface). The analysis becomes highly nonlinear due to the variation in viscosity, $\mu(T)$, and thermal conductivity, $\kappa(T)$, as supposed in (1). The physical model and the coordinate axes are explained in Fig. 1.

Under the above mentioned conditions, the governing equations can be written as:

$$\frac{\partial\left(\hat{r}\hat{u}\right)}{\partial\hat{x}} + \frac{\partial\left(\hat{r}\hat{v}\right)}{\partial\hat{y}} = 0 \tag{3}$$

$$\hat{u}\frac{\partial\hat{u}}{\partial\hat{x}} + \hat{v}\frac{\partial\hat{u}}{\partial\hat{y}} = -\frac{1}{\rho}\frac{\partial\hat{p}}{\partial\hat{x}} + \frac{1}{\rho}\nabla.(\mu\nabla\hat{u}) + g\beta(T - T_\infty)\cos\varphi \tag{4}$$

$$\hat{u}\frac{\partial\hat{v}}{\partial\hat{x}} + \hat{v}\frac{\partial\hat{v}}{\partial\hat{y}} = -\frac{1}{\rho}\frac{\partial\hat{p}}{\partial\hat{y}} + \frac{1}{\rho}\nabla.(\mu\nabla\hat{v}) - g\beta(T - T_\infty)\sin\varphi \tag{5}$$

$$\hat{u}\frac{\partial T}{\partial\hat{x}} + \hat{v}\frac{\partial T}{\partial\hat{y}} = \frac{1}{\rho c_p}(\nabla.(\kappa\nabla T) - \nabla.q_r) \tag{6}$$