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Finite-time stabilization of a class of cascade nonlinear switched systems under state-dependent switching

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ABSTRACT

This paper investigates finite-time stabilization (*FTS*) problem for a class of cascade nonlinear switched systems. Unlike the existing approach based on time-dependent switching strategy, in which the switching instants must be given in advance, largest region function strategy, i.e., state-dependent switching strategy, is adopted to design the switching signal. Based on multiple Lyapunov-like functions method, sufficient conditions for *FTS* of cascade nonlinear switched systems are derived and the corresponding sliding motion problems are also considered. Finally, an example is given to illustrate the applicability and the effectiveness of the proposed method.

1. Introduction

As an important class of hybrid systems, switched systems consist of a finite number of subsystems and a logical rule that orchestrates switching between these subsystems [1]. Due to the success in practical applications [2,3], switched systems have been attracting considerable attention during the last decades. For recent progress, readers can refer to survey papers ([4–6] and the references therein).

Cascade nonlinear switched systems are an important class of hybrid systems, which have broad applications in engineering practice. Switching control of many industry produce processes, switching control of power converters systems, the planar Cartesian haptic display system and so on can be described by such kind of switched systems. Because of the inherent hybrid characteristic of switched systems, the complexity of nonlinear systems, and the interaction of the states of cascade systems, the dynamic behavior of the cascade nonlinear switched systems very complicated. This makes the analysis and synthesis problems for the cascade nonlinear switched systems even more difficult. Few results focusing on such kind of switched systems have been reported up to now. Therefore, the research of cascade nonlinear switched systems has been a hot issue in the control field [7,8]. For such systems, considerable attention has been paid to robust control problem [9], invariance control problem [10], etc. Especially, the L_2-gain disturbance attenuation problem has been concerned in [11–14], and this is a touchy issue since these results usually require solving Hamilton–Jacobi inequalities.

The issues of stabilizability are the basic problems for switched systems and have attracted most of the attention [15–17]. The multiple Lyapunov functions [18] and the single Lyapunov function methods [19] have been proven to be powerful

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and effective tools for finding such a switching signal, which includes time-dependent switching signal [20–22] and state-dependent switching signal [23,24].

Up to now, most of the existing literature related to stability of switched systems focuses on Lyapunov asymptotic stability, which is defined over an infinite time interval. However, in many practical applications, the main concern is the behavior of the system over a fixed finite time interval. Some early results on finite-time stability can be found in [25–27]. The reliable mixed passive and H_∞ filtering problem for uncertain semi-Markov jump delayed systems subject to sensor failures are discussed in [28]. In [29], the problem of extended dissipativity-based state estimation for discrete-time Markov jump neural networks is concerned. In [30], the problem of finite-time control is addressed for a class of interconnected impulsive switched systems with time-varying delay and dynamical disturbance. In [31], the problems of finite-time globally asymptotical stability in probability and finite-time stochastic input-to-state stability for switched stochastic nonlinear systems are investigated. In [32], finite-time boundedness and finite-time stability of switched systems with sector bounded nonlinearity and constant time delay are investigated. The finite-time H_∞ control problem for a class of discrete-time switched singular time-delay with actuator saturation is investigated in [33]. The finite-time stochastic input-to-state stability problem for a class of impulsive switched stochastic nonlinear systems is investigated in [34]. Finite-time boundedness and stabilization problems for a class of switched linear systems with time-varying exogenous disturbances are studied in [35], and a class of state-dependent switching signals are designed such that switched system is finite-time bounded. But in [35], the results on state-dependent switching signals are based on single Lyapunov-like function. Additionally, there are few results available yet finite-time stabilization of cascade nonlinear switched systems, which is quite an important issue for the switched system. This motivates us to carry out present work.

The existing methods of investigating the finite-time stabilization are based on time-dependent switching strategy. There is no available result on *FTS* based on multiple Lyapunov functions for cascade nonlinear switched systems under state-dependent switching signal. Therefore, the main contribution of this paper is that a different approach, largest region function strategy(i.e., state-dependent switching strategy), is adopted to study the problem of *FTS* for cascade nonlinear switched systems. Based on multiple Lyapunov-like functions, some sufficient conditions are given, and the corresponding problem of sliding motion is also considered.

The remainder of the paper is organized as follows. In Section 2, some definitions, assumptions and problem formulations are presented. Section 3 provides the main results of this paper. Some sufficient conditions that can ensure finite-time stabilization of cascade nonlinear switched systems are proposed. Then, an example is presented to illustrate the efficiency of the proposed method in Section 4. Conclusions are given in Section 5.

Notation: the notations used in this paper are standard. Let $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{Z}^+$ denote the field of real numbers, the set of non-negative reals, and the set of non-negative integers, respectively. The notation $P > 0$ means that P is a real symmetric and positive definite; the symbol "$*$" within a matrix represents the symmetric term of the matrix; the super script "T" stands for matrix transposition; $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ refer to, respectively, the n-dimensional Euclidean space and the set of all $m \times n$ real matrices; $\|\cdot\|$ refers to the Euclidean vector norm; I and 0 represent the identity matrix and a zero matrix, respectively; $diag\{\cdots\}$ stands for a block-diagonal matrix. For a square matrix P, $\lambda_{min}(P)$ and $\lambda_{max}(P)$ denote the minimum and maximum eigenvalues of matrix P, respectively.

2. Problem formulation

Consider the following cascade nonlinear switched systems:

$$\begin{cases}\dot{x}_1(t) = A_{1\sigma(t)}x_1(t) + A_{2\sigma(t)}x_2(t) + B_{\sigma(t)}u_{\sigma(t)}\\ \dot{x}_2(t) = f_{2\sigma(t)}(x_2(t))\end{cases} \tag{1}$$

where $x(t) = [x_1^T(t), x_2^T(t)]^T \in \mathbb{R}^n$ is the state, $x_1(t) \in \mathbb{R}^{n-d}$, $x_2(t) \in \mathbb{R}^d$. $u(t) \in \mathbb{R}^m$ is the control input, $\sigma(t) : [0,\infty) \to \zeta = \{1, 2, \ldots, N\}$ is the switching signal which is a piecewise constant function depending on state $x(t)$ in this paper. A_{1i}, A_{2i}, and B_i are constant real matrices for $i \in \zeta$. $f_{2i}(x_2(t))$ is a smooth and continuous function. In this paper, we assume B_i, $i \in \zeta$, is full column rank.

The following switching state feedback controller

$$u_{\sigma(t)} = k_{\sigma(t)}x_1(t) \tag{2}$$

is designed to stabilize system (1). Substituting Eq. (2) into cascade nonlinear switched system (1), we can obtain the closed-loop system as follows:

$$\begin{cases}\dot{x}_1(t) = (A_{1\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)})x_1(t) + A_{2\sigma(t)}x_2(t) = \tilde{A}_{1\sigma(t)}x_1(t) + A_{2\sigma(t)}x_2(t)\\ \dot{x}_2(t) = f_{2\sigma(t)}(x_2(t))\end{cases} \tag{3}$$

The following assumptions for cascade nonlinear switched system (1) are introduced.

Assumption 1 (Du et al. [35]**).** The trajectory $x(t)$ is everywhere continuous, i.e., the state of the cascade nonlinear switched system does not jump at the switching instants.

Assumption 2. The function $G(x_2(t))$ satisfies the constraint $G(x_2(0)) - G(x_2(T_f)) \leq d, \quad d \geq 0$.