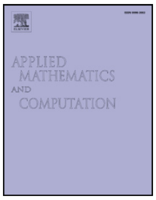
Applied Mathematics and Computation 289 (2016) 258–271

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc



Numerical study on the mechanical response of brain under the impact loading based on elastic–viscoelastic model

Yong Fu [a,b], Qihong Fang [a,b,*], Jia Li [a,b]

[a] State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha 410082, Hunan Province, PR China
[b] College of Mechanical and Vehicle Engineering, Hunan University, Changsha 410082, Hunan Province, PR China

ARTICLE INFO

Keywords:
Brain injury
Elastic–viscoelastic model
Impact loading
Correspondence principle

ABSTRACT

The mechanical response of the brain on the impact process, and the relationship of brain injury and load are investigated rarely due to the complexity of brain. An elastic–viscoelastic model for analyzing the brain impact injury is presented. The elastic–viscoelastic correspondence principle is used to obtain the stress in the brain under impact. This mechanical model is in a good agreement with experimental results, and it can be applied to simulate and analyze brain injury. The results show that the stress of the brain is estimated under the different impact loads. The maximum stress of the brain may reduce by one order of magnitude, whether someone wears the safety helmet or not. Based on the interesting and rational mechanical model, it can be applied in designing the reasonable thickness of safety helmet for protecting brain and provide theoretical basis to determine injury index.

1. Introduction

In recent years, brain injury has attracted much attention due to its vulnerable damaged part in the body [1]. Depending on the statistical data analysis, the reported probability of brain injury is 54% and death rate is 68% in traffic accidents [2]. Meanwhile, brain injury caused by the explosion or blast is most common in the war [3]. Brain injury is often attributed excessive impact force or inertial force on the head [4]. Motivated by the high incidence and mortality of brain injury, some efforts have been devoted to study the bio-mechanics of brain injury [5,6]. However, little work focuses on studying the mechanical response to brain tissue on the impact and the relationship between brain injury and impact load. It can help us understand the damage mechanism of brain injury, and reduce brain injury.

There are the three ways to study the bio-mechanics of brain impact injury, such as experimental methods [7–9], theoretical models [10–13], and numerical simulations [14–16]. Biomechanists always construct a particular system, and get the physical and geometric properties. The impact loading is applied as a concrete input. The mechanical response as output is determined by the means of experiments, theoretical models and numerical simulations. Recent developments on the quantitative characteristics are that biomechanists are permitted to study the difference of brain injury in adults and

---

* Corresponding author at: College of Mechanical and Vehicle Engineering, Hunan University, Changsha 410082, Hunan Province, PR China. Tel.: +86 73188823517; fax:+86 73188822330.

E-mail address: fangqh1327@hnu.edu.cn (Q. Fang).

http://dx.doi.org/10.1016/j.amc.2016.05.005

Nomenclature

F	concentrated force
h	thickness of hard material
σ_{z1}	stress on the hard material
$\sigma_{z1(h)}$	stress on the interface S
P	distributed load
dS	infinitesimal area
σ_{z2}	stress on soft material
μ	Poisson's ratio
E_1	Young modulus of brain tissue
E_2	Young modulus of cerebrospinal fluid
η_1	coefficient of the viscoelastic material
t	time
$\bar{S}_{ij}$	Laplace transform of deviator stress tensor
$\bar{e}_{ij}$	Laplace transform of deviator strain tensor
$\bar{\sigma}_{kk}$	Laplace transform of spherical stress tensor
$\bar{\varepsilon}_{kk}$	Laplace transform of spherical strain tensor
$\bar{P}', \bar{Q}'$	Differential operators of the deviator tensor
$\bar{P}'', \bar{Q}''$	differential operators of the spherical tensor
p'_k, q'_k, p''_k, q''_k	constants of the viscoelastic material
K	bulk modulus
G	shear modulus
H(t)	step function
υ	kinematic viscosity
ρ_0	density

infants [17–21]. The pathological and physiological indexes of experimental models and the responses of these models can be applied to the mechanical analysis models. These indexes and responses as inputs and validations can serve to analyze the mechanism of brain injury. The related indexes can be obtained under the condition of impact, and reliable conclusions of the brain injury can be drawn in this approach.

At present, it cannot be measured that a minor change under stress variation or strain changes of brain tissues. It is required to establish the mechanical analysis model to analyze the mechanism of brain injury. Moreover, as the complexity of the brain in structures and material, it is difficult to simulate the mechanical response to the impact load to the brain tissues. Therefore, some simplifications are needed. Holbourn [22] had set up the bio-mechanical model of brain injury by the theory of elasticity. Since the 1960s, a number of scholars had been developed more viscoelastic models of brain injury in the ensuing decades [23–25]. During the history of studying brain injury, the material of the skull is viewed as rigid, elastic and viscoelastic material [10,26–28]. The material of brain tissue is modeled as inviscid liquid, viscoelastic media in recent years [29,30].

This paper presents an interest mechanical model—a hard material based on soft substrates, and extends it to analyze the brain impact injury. The skull is simplified to hard elastic material, while the brain tissue is simplified with soft viscoelastic material. According to the mechanical model, the mechanical response to brain tissue on the impact and the relationship between brain injury and impact load can be obtained. Furthermore, it could provide the theoretical basis of brain injury prevention and the standard of brain injury.

2. Model description

Based on the experimental date of Zhang et al. [31], the soft material is used as substrate and the hard material above the soft material, as shown in Fig. 1. A force F is applied at the origin O of the hard material along the direction of the z-axis. The thickness of hard material is set as h. The subscripts 1 and 2 represent the hard material and soft material, respectively. Using this model to analyze brain injury, skull can be regarded as hard material due to the hardness of skull higher than that of brain tissues, and brain tissues can be treated as soft material. Hard material can be seen as elastic-plastic material, and soft material is considered as viscoelastic material. The impact loading is treated as the concentrated force F, which acts on the hard material. Owing to the structure of the brain is very complex in the analysis of brain injury, so the model of the brain is simplified as follows (See, Fig. 1).