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A simple mathematical model of microtubules surrounded by an elastic matrix by nonlocal finite element method

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ABSTRACT

A simple nonlocal beam model is proposed to study buckling response of protein microtubules. The size-effect for buckling model of microtubules is considered by using the nonlocal continuum theory. Finite element procedure is used for solution of nonlocal differential equation of microtubules for elastic stability. The influence of the small length scale on the buckling value is examined for different geometric parameters. The effect of elastic matrix surrounded of microtubules is also examined and some benchmark results are presented.

1. Introduction

Nano- and micro-scaled biological and mechanical systems hold and important research field for the many disciplines. Nanowires, nanobridges, nanotubes, micro- and nanosensors, atomic force microscope, and micro/nano-electromechanical devices are widely used in different engineering and industrial applications [1–7]. A continuum model is generally used for modeling by researchers of such systems due to its computational efficiency and simplicity than the atomic modeling. So, different types of size dependent continuum theory are developed and used for modeling. Cosserat elasticity, couple stress theory, strain gradient elasticity, surface energy theory, stress gradient theories are generally used such a nano and micros system modeling. Nonlocal continuum theory proposed by Eringen [8] is also widely used in the past ten years. Unlike the classical elasticity (Cauchy) theory, the nonlocal elasticity theory assumes that the stress at a point of any continuum body is a function of strains at all points in the given continuum body. By this time, many researchers used this theory for modeling of nano- or micro-scaled mechanical or biological systems based on the rod, beam or plate theories [9–23].

The classical elasticity theory is not suitable for modeling of such type micro-sized or nano-sized biological or mechanical systems. It is concluded that by some experimental and theoretical studies, length scale parameters or size effects play a major role in mechanical behavior of nano- and micro-scaled systems [24–34]. As mentioned before, classical elasticity theory does not take into consideration the size effect of microstructure during the formulation [35–41]. In order to introduce the size effect to the governing equations, material length scale parameters must be taken into account. Atomistic and molecular dynamic simulation models or hybrid atomistic-continuum models are computationally expensive. Furthermore, controlled experimental studies are very difficult task for nano-scale devices in many conditions.

Microtubules (MT) are composed of two-different tubulin as alfa and beta and have an important function in the cell. They are the main components of cytoskeleton [42–48]. Microtubules are proteins organized in a network that is interconnected with microfilaments and intermediate filaments to form the cytoskeleton structures [49–54]. Microtubules are

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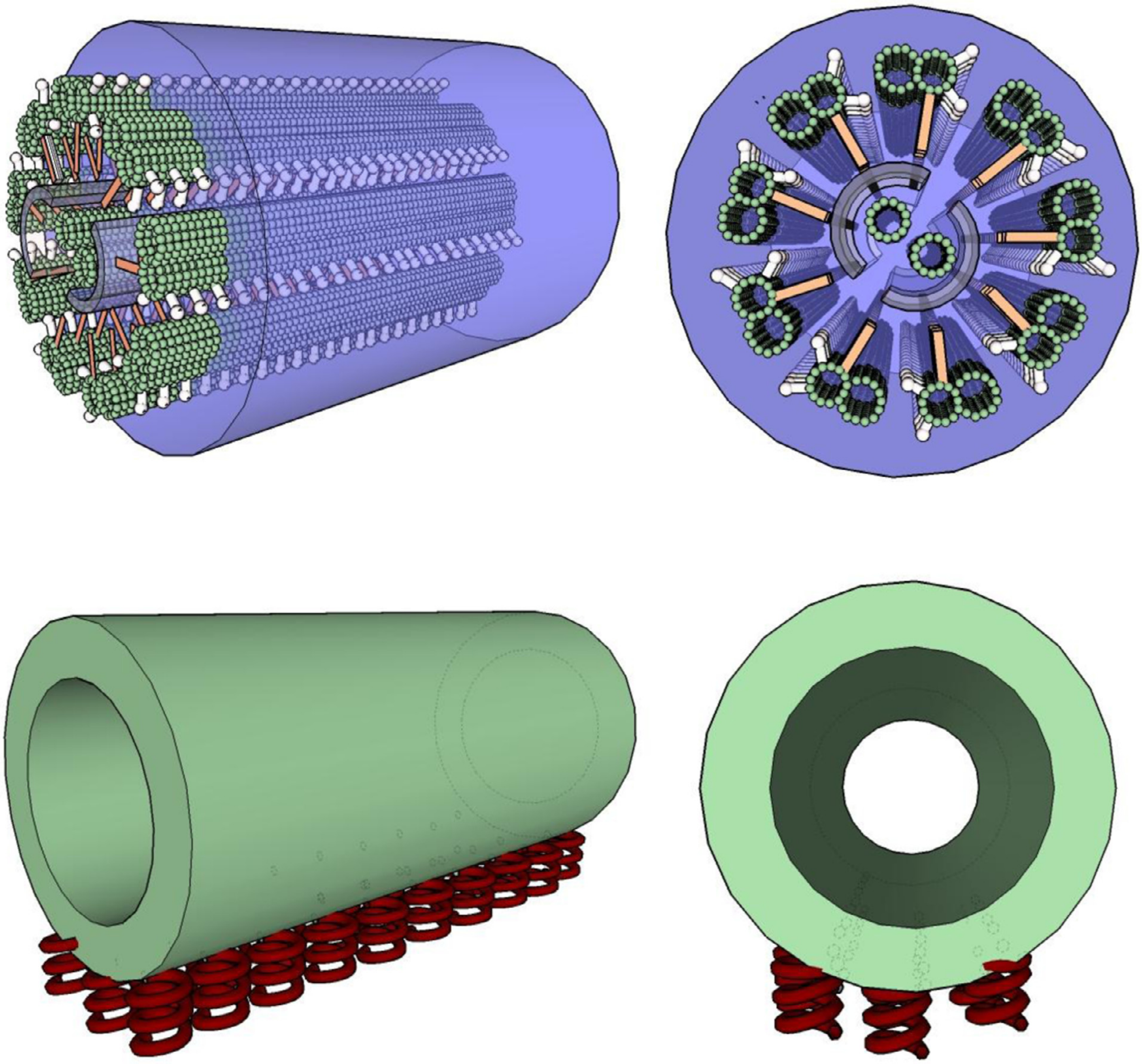

Fig. 1. A typical protein microtubules surronded and elastic matrix.

cylindrical tubes. The protein subunits are bound laterally into protofilaments. The mechanical properties of microtubules play an important role in process such as cell division and intracellular transport [55–67]. Microtubules are the most rigid of the cytoskeleton filaments. As a result, the mechanical properties of these structures have been widely studied in the past years. There have been great deals of experimental and theoretical research in recent past years related the mechanical properties of biological systems [68–83].

The structure of microtubules is cylindrical and it typically involves 13 or 15 parallel protofilaments which are connected laterally into hollow tubes as shown in Fig. 1. MTs are considered as hollow cylinders having 25 nm external and 15 nm internal diameters [43–48]. The length of MTs can vary from tens of nanometers to hundreds of microns. Furthermore, MTs are considered as self-assembling biological nanotubes that are essential for cell motility, building the cytoskeleton, cell division and intracellular transport. The average Young's modulus of microtubules is $\sim$ 2.0 GPa [42–53]. Among the three types of cytoskeleton filaments, microtubules are the most rigid. The bending rigidity of microtubules is about 100 times that of intermediate and actin filaments. As parallel to rapid developments in nanotechnology and computational methods, some mechanical models have applied for analysis of MTs [46–54]. In the past ten years, higher-order continuum theories such as nonlocal elasticity, couple stress elasticity or strain gradient elasticity theory have been used for different engineering applications.