



Jain–Durrmeyer operators associated with the inverse Pólya–Eggenberger distribution



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ABSTRACT

The present paper deals with the Jain–Durrmeyer operators based on inverse Pólya–Eggenberger distribution. First, we give the moments with the help of Vandermonde convolution formula and then study approximation properties of these operators which include uniform convergence and degree of approximation.

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1. Introduction

The Pólya–Eggenberger (P-E) distribution was introduced by Eggenberger and Pólya [11] in the year 1923. The P-E distribution with parameters (n, A, B, S) is defined as:

$$P(X = k) = \binom{n}{k} \frac{\prod_{i=0}^{k-1} (A + iS) \prod_{i=0}^{n-k-1} (B + iS)}{\prod_{i=0}^{n-1} (A + B + iS)}, \quad k = 0, 1, \dots, n. \quad (1.1)$$

This gives the probability of getting k white balls out of n drawings from an urn contains A white and B black balls, if each time one ball is drawn at random and then replaced together with S balls of the same color.

In literatures, the inverse Pólya–Eggenberger(I-P-E) distribution is defined as:

$$P(X = k) = \binom{n+k-1}{k} \frac{\prod_{i=0}^{k-1} (A + iS) \prod_{i=0}^{n-1} (B + iS)}{\prod_{i=0}^{n+k-1} (A + B + iS)}, \quad k = 0, 1, \dots, n, \quad (1.2)$$

gives the probability that k white balls are drawn preceding the n -th black ball. The details have been given about these two distributions (1.1) and (1.2) in [21].

Based on I-P-E (1.2), Stancu [28] studied a generalization of the Baskakov operators for a real-valued function bounded on $[0, \infty)$, defined as:

$$V_n^{(\alpha)}(f; x) = \sum_{k=0}^{\infty} v_{n,k}^{(\alpha)}(x) f\left(\frac{k}{n}\right) = \sum_{k=0}^{\infty} \binom{n+k-1}{k} \frac{1^{[n, -\alpha]} x^{[k, -\alpha]}}{(1+x)^{[n+k, -\alpha]}} f\left(\frac{k}{n}\right) \quad (1.3)$$

where α is a non-negative parameter which may depend only on the natural number n and $t^{[n, h]} = t(t-h)(t-2h) \dots (t - (n-1)h)$, $t^{[0, h]} = 1$ represents the factorial power of t with increment h . In the case when $\alpha = 0$, operators (1.3) reduce

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to the following classical Baskakov operators [6].

$$V_n(f, x) = \sum_{k=0}^{\infty} \binom{n+k-1}{k} \frac{x^k}{(1+x)^{n+k}} f\left(\frac{k}{n}\right). \quad (1.4)$$

In the year 1972, Jain [20] introduced a new class of linear operators (known as Jain operators) as:

$$J_n^{(\beta)}(f; x) = \sum_{k=0}^{\infty} b_{n,k}^{(\beta)}(x) f\left(\frac{k}{n}\right), \quad x \geq 0, \quad (1.5)$$

where $0 \leq \beta < 1$ and

$$b_{n,k}^{(\beta)}(x) = \frac{nx(nx+k\beta)^{k-1}e^{-(nx+k\beta)}}{k!}.$$

These operators reduce to Szász–Mirakyan operators for special case $\beta = 0$. Several mathematicians studied these Jain operators and their integral variant (See [3,4,12,15,25,29]). Very recently, Gupta and Greubel [16] introduced the Durrmeyer variant of these operators (1.5) in a very different manner as:

$$\begin{aligned} D_n^{(\beta)}(f; x) &= \sum_{k=0}^{\infty} \left(\int_0^{\infty} b_{n,k}^{(\beta)}(t) dt \right)^{-1} b_{n,k}^{(\beta)}(x) \int_0^{\infty} b_{n,k}^{(\beta)}(t) f(t) dt \\ &= \sum_{k=0}^{\infty} \frac{\langle b_{n,k}^{(\beta)}(t), f(t) \rangle}{\langle b_{n,k}^{(\beta)}(t), 1 \rangle} b_{n,k}^{(\beta)}(x), \end{aligned} \quad (1.6)$$

where $\langle f, g \rangle = \int_0^{\infty} f(t)g(t)dt$.

These operators also reduce to Szász–Mirakyan Durrmeyer operators for $\beta = 0$. Moreover, the estimation of moments for (1.6) has been calculated by using Stirling numbers of first kind and confluent hypergeometric function. Durrmeyer form of different operators have been studied by many mathematicians and researchers (See [7,8,22]). Motivated from [16], now we propose Hybrid type operators as:

$$\begin{aligned} L_n^{(\alpha)}(f; x) &= \sum_{k=0}^{\infty} \left(\int_0^{\infty} b_{n,k}^{(\alpha)}(t) dt \right)^{-1} v_{n,k}^{(\alpha)}(x) \int_0^{\infty} b_{n,k}^{(\alpha)}(t) f(t) dt \\ &= \sum_{k=0}^{\infty} \frac{\langle b_{n,k}^{(\alpha)}(t), f(t) \rangle}{\langle b_{n,k}^{(\alpha)}(t), 1 \rangle} v_{n,k}^{(\alpha)}(x). \end{aligned} \quad (1.7)$$

For $\alpha = 0$, these operators include the hybrid operators (Baskakov–Szász–Durrmeyer operators) presented in [1,18] etc.

These days several mathematicians have worked on P-E based positive linear operators [2,13,17,24,26,27] and few mathematicians have discussed I-P-E based positive linear operators [14,19,28]. Very recently Deo et al. [9] defined I-P-E based Baskakov–Kantorovich operators and studied its properties, asymptotic formula as well as degree of approximation.

Now in the present paper, we obtain the moments of I-P-E distribution based operators (1.3) by using Vandermonde convolution formula, however these moments have already been calculated by Stancu in [28] by using hypergeometric series. Then we consider Jain–Durrmeyer operators associated with the inverse Pólya–Eggenberger distribution (1.7) and study uniform convergence and degree of approximation for these operators.

2. Auxiliary results

Let \mathbb{N} be the set of natural numbers and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. For $k \in \mathbb{N}_0$, let $e_k(x) = x^k$ be the test functions. The computation of the images of test functions by Stancu–Baskakov operators (1.3) was done in [28]. Now we find those images with the help of Vandermonde convolution formula and following relation:

$$t^{[i+j,h]} = t^{[i,h]} \cdot (t - ih)^{[j,h]} \quad (2.1)$$

for $i, j \in \mathbb{N}$ and $h \neq 0$.

Lemma 2.1. For the Stancu–Baskakov operators (1.3), the following results hold:

$$V_n^{(\alpha)}(e_0; x) = 1, \quad V_n^{(\alpha)}(e_1; x) = \frac{x}{1-\alpha},$$

and

$$V_n^{(\alpha)}(e_2; x) = \frac{1}{(1-\alpha)(1-2\alpha)} \left[x^2 + \frac{x(x+1)}{n} + \alpha \left(1 - \frac{1}{n} \right) x \right].$$

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