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Robust control of uncertain semi-Markovian jump systems using sliding mode control method

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ABSTRACT

The problem of robust adaptive sliding mode control for semi-Markovian jump systems with actuator faults is investigated in this paper. The uncertainties considered in this paper satisfy norm-bounded form, and bounds of nonlinearity, actuator faults and external disturbance are unknown. Then, the influences of the actuator faults, unknown nonlinearity and disturbance can be effectively attenuated via a novel adaptive sliding mode controller. The reachability of sliding mode surface can be guaranteed by the adaptive sliding mode controller. Using Lyapunov stability theory, sufficient conditions are derived to guarantee the stochastic stability of the sliding mode dynamics. Finally, a numerical example is exploited to demonstrate the effectiveness of the proposed method.

1. Introduction

Markovian jump systems (MJSs) are one special type of stochastic switching systems. MJSs exist in the complex practical dynamical systems subject to abrupt structure variations. Recently, many results about modeling, stability, control and filter design for MJSs have been reported [1–16]. The authors in [4] designed a controller to stabilize Markovian jump linear system over networks with random communication delay. The authors in [9] considered the robust extended dissipative control problem for sampled-data MJSs. The jump time of system mode obeys exponential distribution in MJSs, which implies the transition rate has no connection with jump time. The transition rates of semi-Markovian jump systems (S-MJSs) are related to jump time, which are different from the constant transition rates in MJSs. Thus, S-MJSs are more practical than MJSs in modeling practical dynamical systems since S-MJSs are less conservative. More recently, the stochastic stability analysis and stabilization for S-MJSs are studied in [17,18]. Moreover, the probability distribution of sojourn time of a Markov chain from exponential distribution to Weibull distribution was investigated in [19].

In practical systems, however, the actuator failure may occur. Thus, it may cause instability of the control systems and deteriorate the system performance. Recently, many results on the control systems with partial actuator faults have been reported [3,20–25]. For example, the problem of event-triggered based fault detection for nonlinear networked control systems was studied in [26]. To address the systems with sensor fault, the authors in [27] proposed an observer based fault detection scheme.

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For the uncertain systems, some advanced control strategies have been proposed, including fuzzy control [28–31], adaptive control [32–36] and sliding mode control (SMC) [37]. For the uncertain nonlinear networked control systems, an interval type-2 observer-based control scheme was proposed in [38]. An adaptive optimal control strategy was provided for a category of nonlinear uncertain systems with dead zone in [39]. Wang et al in [40] applied the adaptive control design to fully actuated marine surface vehicles. Among these approaches, SMC has been paid considerable attention [41–45]. SMC algorithm has been employed to a series of dynamical systems in the past few decades [46,47]. Generally speaking, there are two basic steps in the design of SMC. First, we construct an appropriate sliding mode surface to make the resulting sliding mode dynamics have some satisfactory properties. Second, we design an SMC controller to drive the system state trajectories onto the predefined sliding mode surface. More recently, some remarkable results on the SMC problem have been achieved in [3,48–58]. To mention a few, the authors investigated the design of SMC subject to packet losses in [54], which didn't consider uncertainties, unknown nonlinearities and unknown external disturbance. Using interval type-2 fuzzy approach to model the considered system, an adaptive sliding mode control was proposed in [59]. Li et al in [60,61] provided an output-feedback based adaptive sliding model control strategies for fuzzy systems with actuator saturation and nonlinear Markovian jump systems. The authors in [56] studied the SMC problem of nonlinear singular stochastic systems with Markovian switching parameters, but not considering uncertainties, unknown nonlinearities and disturbance. Besides, the authors in [56] did not use weighted sum approach to address different input matrix. The authors in [62] considered the state estimation and sliding mode control problems for phase-type S-MJSs and designed a novel sliding mode controller instead of considering nonlinearities and unknown external disturbances. However, the actuator degradation and external disturbance are not considered in this paper. In addition, the bounds of nonlinearity, actuator faults and external disturbance are unknown, which motivates this study.

The aim of this paper is to investigate the problem of adaptive SMC for S-MJSs with parameter uncertainties, disturbance, nonlinearity and actuator faults. First, an appropriate common sliding mode surface is constructed by a weighted sum method such that the resulting reduced-order systems are stochastically stable. Second, the designed adaptive sliding mode controller can guarantee the reachability. Finally, a numerical example is provided to show the effectiveness of the proposed scheme. The organization of this paper is given as follows. The main problems are formulated in Section 2. Section 3 designs a common sliding mode surface and Section 4 designs an adaptive sliding mode controller. Section 5 provides the reachability of sliding mode surface and Section 6 presents a numerical example to demonstrate the validity of the mentioned method, and finally Section 7 gives the conclusion.

Notations: The superscript "T" represents the matrix transposition, $\mathbb{R}^n$ shows the n -dimensional Euclidean space. The notation $X > 0$ means that X is real symmetric and positive definite. $\|\cdot\|_1$ and $\|\cdot\|_2$ refer to the 1-norm and usual Euclidean vector norm, respectively. The notation diag$(\cdot)$ denotes a diagonal matrix. The vector $e_i \in \mathbb{R}^s$ is the ith standard basic vector; and 1_s is consisted of ones; $\lambda_{min}(P)$ and $\lambda_{max}(P)$ stand for the minimum and maximum eigenvalue of a real symmetric matrix P, respectively. The notation $\mathbf{E}\{\cdot\}$ represents the mathematical expectation operator. $\bigotimes$ represents the Kronecker product. The notations tr$(\cdot)$ and sgn$(\cdot)$ stand for the trace and sign function, respectively. The notation $(\Omega, \mathcal{F}, \mathcal{P})$ denotes the probability space. Ω, $\mathcal{F}$ and $\mathcal{P}$ represent the sample space, σ-algebra of subsets of the sample space and probability measure on $\mathcal{F}$, respectively. The symbol "$*$" represents a term that is induced by symmetry.

2. Problem formulation

Consider the S-MJS in the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ as follows:

$$\dot{x}(t) = (A(r_t) + \triangle A(r_t))x(t) + B(r_t)(u^F(t) + f(x,t) + d(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the state, $u^F(t) \in \mathbb{R}^m$ stands for fault control input, $f(x,t) \in \mathbb{R}^m$ is nonlinearity and $d(t) \in \mathbb{R}^m$ denotes disturbance. $A(r_t) \in \mathbb{R}^{n\times n}$ and $B(r_t) \in \mathbb{R}^{n\times m}$ are constant system matrices, respectively. $\triangle A(r_t) \in \mathbb{R}^{n\times n}$ indicates the system uncertainties. $\{r_t,\ t \geq 0\}$ represents a continuous-time semi-Markovian process with taking values in a discrete set $S = \{1, 2, \dots, s\}$. The transition rate matrix $\Pi = (\pi_{ij})_{s\times s}$ is given as

$$\Pr\{r(t+l) = j \mid r(t) = i\} = \begin{cases} \pi_{ij}(l)l + o(l), & i \neq j, \\ 1 + \pi_{ii}(l)l + o(l), & i = j, \end{cases}$$

where $l > 0$, $\lim_{l\to 0} \frac{o(l)}{l} = 0$, and $\pi_{ij}(l) \geq 0$ denotes the transition rate from mode i to mode j for $i \neq j$, and $\pi_{ii}(l) = -\sum_{j=1,\ j\neq i}^{s} \pi_{ij}(l)$.

Remark 1. The probability distribution of sojourn time is exponential distribution in MJS, which is distinguished from S-MJS [63,64]. Its probability distribution of sojourn time is Weibull distribution such that the transition rate is time varying. In practice, the time-varying transition rate $\pi_{ij}(l)$ is generally bounded as $\underline{\pi}_{ij} \leq \pi_{ij}(l) \leq \bar{\pi}_{ij}$, where $\underline{\pi}_{ij}$ and $\bar{\pi}_{ij}$ are real constant scalars, respectively.

Considering ith mode $(i = 1, 2, \dots, s)$, we represent S-MJS (1) in the following form:

$$\dot{x}(t) = \left(A_i + \triangle A_i\right)x(t) + B_i\left(u^F(t) + f(x,t) + d(t)\right), \tag{2}$$