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Approximate controllability for stochastic evolution inclusions of Clarke's subdifferential type

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ABSTRACT

In this paper, we deal with the approximate controllability of stochastic evolution inclusions of Clarke's subdifferential type. Firstly, by using stochastic analysis, nonsmooth analysis, theory of operator semigroups and fixed point theorems of multivalued maps, we show the existence of mild solutions for the stochastic evolution inclusions. Then we provide a sufficient condition to guarantee the approximate controllability of the stochastic evolution inclusions. Actually, our results cover a broader class of inclusion problems involving time depending operators. Finally, an example is included to illustrate the applicability of the main results.

1. Introduction

In this paper, we study the existence of mild solutions and approximate controllability of the following stochastic evolution inclusions of Clarke's subdifferential type involving time depending operator:

$$\begin{cases} dx(t) \in (A(t)x(t) + Bu(t))dt + \partial F(t, x(t))dt + \sigma(t, x(t))dw(t), \quad t \in J = [0, b], \\ x(0) = x_0, \end{cases} \tag{1.1}$$

where $x(\cdot)$ takes values in the separable Hilbert space H, $\{A(t): t \in J\}$ is a family of linear closed densely defined operators on H such that the domain of $A(t)$ does not depend on t. The control function $u(\cdot)$ taking values in a separable Hilbert space U and B is a bounded linear operator from U into H. The notation ∂F stands for the Clarke generalized subdifferential (cf. [6]) of a locally Lipschitz function $F(t, \cdot)$: $H \to R$. σ is a given appropriate functions to be specified later. w is a Q-Weiner process on a complete probability space $(\Omega, \Gamma, \mathbb{P})$ and x_0 is Γ_0 measurable H-valued random variables independent of w.

It is well known that the control theory is an important area of application oriented mathematics which deals with the design and analysis of control systems. In recent years, controllability problems for various types of nonlinear dynamical systems have been considered by using different kinds of approaches in many publications. Further, there have been increasing interests in the study of the controllability problems for stochastic systems due to its applications in economics, ecology and finance and so on. Currently, fruitful achievements have been obtained on the controllability of stochastic systems and

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inclusion problems, see e.g. Bashirov and Mahmudov [4], Mahmudov [23], Obukhovski and Zecca [28], and Rykaczewski [33] and the references therein. Also, there are many interesting results on the theory and applications of stochastic differential equations. More precisely, Arthi et al. [1,2] consider the existence and controllability impulsive stochastic evolution systems with state-dependent and infinite delay. Klamka [13,14] studied the stochastic controllability systems with delays. Lin and Hu [15] considered the existence results of stochastic inclusions with nonlocal initial conditions. Sakthivel et al. [37,39,40] obtained the approximate controllability of semilinear fractional differential systems in Hilbert spaces. Ren et al. [34] researched the controllability of impulsive neutral stochastic differential inclusions with infinite delay. Relevant results regarding the approximate controllability for stochastic systems can be found in [3–5,18,35,41,42,45,46] and the references therein.

Recently, many researchers have paid increasingly attention to the evolution inclusion problems of Clarke subdifferential type which has important applications in mechanics and engineering, especially in nonsmooth analysis and optimization (see [6,25]). It is effectively in mathematical models to describe the frictional contact between a piezoelectric body and an electrically conductive foundation [26], and to describe the antiplane shear deformations of a piezoelectric cylinder in frictional contact with a foundation [27]. Moreover, hemivariational inequalities represent a class of nonlinear inclusions that are associated with the Clarke subdifferential operator (see [17,19,24]). Hemivariational inequality was introduced to deal with the mechanical problems with nonsmooth and nonconvex energy superpotentials (see [30,31]). Therefore, it is necessary and important to study evolution inclusions with Clarke's subdifferential type. For more details can be found in the papers [20,43,44] and the references therein.

At present, although some significant results have been obtained for the solvability and control problems of evolution inclusion problems. We mention that recently the researchers concerted with the problems containing $A(t) \equiv A$ for approximate controllability of fractional stochastic differential inclusions with nonlocal conditions [38], fractional stochastic partial neutral integro-differential inclusions with infinite delay [47]. And the existence and completely controllable results, which related to Clarke's subdifferential type, for stochastic evolution inclusions [16] and fractional stochastic evolution hemivariational inequality [21] are obtained. However, the approximate controllability of the systems described by stochastic evolution inclusions of Clarke's subdifferential type involving time depending operator has not been investigated yet and the investigation on this topic has not been appreciated well enough. Motivated by above consideration, we will consider the existence of mild solutions and approximate controllability of stochastic evolution inclusions (1.1) of Clarke's subdifferential type with time-depending operators $A(t)$ which is a family of linear closed densely operators. Furthermore, our results are different form the works of [16,21,38,47] and the construction is effective giving explicit expressions for the involved approximate controls and corresponding states.

The rest of this paper is organized as follows. In Section 2, we will recall some basic definitions and useful preliminary facts that will be needed in the sequel. In Section 3, the existence of mild solutions for the stochastic evolution inclusions are established and proved by applying stochastic analysis techniques, semigroup of operators theory, a fixed point theorem of multivalued maps and properties of Clarke generalized subdifferential. In Section 4, the approximate controllability of the system (1.1) is formulated and proved mainly by using a fixed point technique. Finally, an example is given to illustrate our main results in Section 5.

2. Preliminaries

Let $(\Omega, \Gamma, \{\Gamma_t, t \geq 0\}, \mathbb{P})$ be a complete probability space equipped with a normal filtration $\{\Gamma_t,\ t \geq 0\}$ satisfying that Γ_0 contains all $\mathbb{P}$-null sets of Γ. $E(\cdot)$ denotes the expectation of a random variable or the Lebesgue integral with respect to the probability measure $\mathbb{P}$. Let H, U be the separable Hilbert spaces and $\{w(t), t \geq 0\}$ is a Wiener process with the linear bounded covariance operator Q such that $\mathrm{tr}Q < \infty$.

We suppose that there exist a complete orthonormal system $\{e_k\}_{k \geq 1}$ in H, a bounded sequence of nonnegative real numbers λ_k such that $Qe_k = \lambda_k e_k$ $(k = 1, 2, \ldots)$ and a sequence of independent Brownian motions $\{\beta_k\}_{k \geq 1}$ such that

$$\langle w(t), e\rangle = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\langle e_k, e\rangle \beta_k(t), \quad e \in H, \quad t \geq 0,$$

and $\Gamma_t = \Gamma_t^w$, where Γ_t^w is the σ-algebra generated by $\{w(s) : 0 \leq s \leq t\}$.

Let $L_0^2 = L^2(Q^{\frac{1}{2}}H, H)$ be a space of all Hilbert-Schmidt operators from $Q^{\frac{1}{2}}H$ to H with the inner product $\langle \phi, \varphi\rangle_{L_0^2} = \mathrm{Tr}[\phi Q \varphi^*]$. $L^2(\Gamma, H) = L^2(\Omega, \Gamma, \mathbb{P}, H)$ denotes a Hilbert space of strongly Γ-measurable, H valued random variables satisfying $E\|x\|_H^2 < \infty$. Since for each $t \geq 0$ the sub-σ-algebra Γ_t is complete, $L^2(\Gamma_t, H)$ is a closed subspace of $L^2(\Gamma, H)$, and hence $L^2(\Gamma_t, H)$ is a Hilbert space. Let $L^2(\Gamma_b, H)$ be a Banach space of all Γ_b-measurable square integrable random variables with values in the Hilbert space H.

Let $C(J, L^2(\Gamma, H))$ be a Banach space of all mean square continuous maps from J into $L^2(\Gamma, H)$ with the norm $\|x\| = (\sup_{t\in J} E\|x(t)\|^2)^{\frac{1}{2}} < \infty$. $L_\Gamma^2(J, H)$ will denote the Hilbert space of all random processes Γ_t-adapted measurable defined on $J = [0, b]$ with values in H with the norm $\|x\|_{L_\Gamma^2(J,H)} = (\int_0^b E\|x(t)\|_H^2)^{1/2} < \infty$. The space $L_\Gamma^2(J, U)$ has the same definition as $L_\Gamma^2(J, H)$ with the norm $\|u\|_{L_\Gamma^2(J,U)} = (\int_0^b E\|u(t)\|_U^2)^{1/2} < \infty$. For details, we refer the reader to [32,36] and references therein.