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Valuation of power option for uncertain financial market

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ABSTRACT

Power option is such an option whose payoff is based on the price of the underlying asset raised to some power. Unlike Black–Scholes setting, we investigate the valuation of power options under the assumption that the underlying stock price is assumed to follow an uncertain differential equation, and derive the pricing formulas of power options for Liu's uncertain stock model with the method of uncertain calculus based on uncertainty theory. Some numerical examples are given to illustrate the pricing formulas.

1. Introduction

Power option depends on the underlying asset price raised to some power. For the financial market participants, this type of option can afford great flexibility and a substantial amount of leverage over ordinary options. Compared with buying an ordinary option, an investor will possibly yield substantially more premium income through buying this type of power option. Due to the characteristic of high leverage, power option has been attracting the attention of both financial institutions and investors, and has been widely used in the financial fields. There have been many examples of the instruments with power option payoff (see Macovschi and Quittard-Pinon [20]), for example, the power options with a power of order 2 have been issued by Bankers Trust in Germany.

Previous studies of pricing power option are mainly with the method of stochastic finance based on the probability theory, and the underlying asset price are usually assumed to follow some stochastic differential equation. The related research can be see the works of Macovschi and Quittard-Pinon [20] and Kim et al. [10]. But many empirical investigations showed that the price of underlying asset does not behave like randomness, and it is often influenced by the belief degrees of investors since investors usually make their decisions based on the degrees of belief rather than the probabilities. For example, one of the key elements in the Nobel-prize-winning theory of Kahneman and Tversky [9,24] is the finding of probability distortion which showed that decision makers usually make their decisions based on a nonlinear transformation of the probability scale rather than the probability itself, people often overweight small probabilities and underweight large probabilities.

The home bias is another puzzle in financial fields, based on the traditional portfolio theory investors should choose the optimal allocation the theory suggested by Sharpe [23] and Lintner [12], but a lot of surveys showed that investors usually overweight the domestic stock markets and companies. Many scholars try to explain the home bias puzzle, for example,

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Ahearne et al. [1], Devereux and Saito [8], Lewis [11] and Ueda [25] gave their explanations. But we argue that investors' belief degrees play an important role in decision making for financial practice.

Previous option pricing methods are mainly within the framework of the pricing option theory of Black–Scholes [2] and Merton [21] in which the underlying asset price process is assumed to follow the stochastic differential equations. In 2013, Liu [17] gave a convincing paradox to show that using any stochastic differential equations to describe the stock price process is inappropriate. This viewpoint can also be demonstrated by the empirical phenomenon that the distribution of underlying asset has a higher peak and heavier tails than normal probability distribution.

For rationally dealing with belief degrees, Liu [13] founded uncertainty theory in 2007. It has become a branch of axiomatic mathematics to deal with belief degrees, and has been applied to many fields successfully, including uncertain programming, uncertain statistics, uncertain risk analysis, uncertain finance and uncertain control and so on. Within the framework of uncertainty theory, there are many scholars devoting themselves to the study of financial problems after Liu's pioneer work of uncertain finance in 2009, such as the pricing formulas of European option, American option and geometric average Asian option for Liu's uncertain stock model were proved by Liu [15], Chen [4] and Zhang and Liu [28], respectively. Uncertain interest rate term structures were discussed by Chen and Gao [5]. A no-arbitrage theorem for this type of uncertain stock model was verified by Yao [27]. Besides, Chen et al. [6] investigated the option pricing problem with periodic dividends and derived the pricing formulas within framework of uncertainty theory. Peng and Yao [22] proposed an uncertain mean-reverting stock model, and presented a pricing method for their model. Liu et al. [19] employed uncertainty theory to study the pricing problem of currency option and derived the related price formulas for their uncertain currency model presented by them. Zhang et al. [29] gave the pricing formulas of interest rate ceiling and floor with the method of uncertain calculus.

In this paper, within the framework of uncertainty theory, we investigate the pricing problem of power option. Based on the assumption that the stock price process follows an uncertain differential equation, we obtain the power option pricing formulas for Liu's uncertain stock model.

The rest of the paper is organized as follows. In next section, some useful concepts and theorems of uncertainty theory as needed are introduced. In Section 3, a brief introduction of uncertain stock model is given. In Section 4, the valuation of power option for uncertain stock model is explored. Finally, a brief conclusion is made in Section 5.

2. Preliminary

As a branch of axiomatic mathematics, uncertainty theory is a useful tool to deal with the problems with belief degrees associated with human uncertainty. For better understanding this paper, some preliminary knowledge of uncertainty theory as needed is introduced as follows.

2.1. Uncertain variable

Definition 2.1 (Liu [13]). Let Γ be a nonempty set, and let $\mathcal{L}$ be a σ-algebra over Γ. An uncertain measure is a function $\mathcal{M}: \mathcal{L} \to [0,1]$ such that

Axiom 1. (Normality Axiom) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ;
Axiom 2. (Duality Axiom) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ;
Axiom 3. (Subadditivity Axiom) For every countable sequence of events $\{\Lambda_i\}$ we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}. \tag{2.1}$$

A set $\Lambda \in \mathcal{L}$ is called an event. The uncertain measure $\mathcal{M}\{\Lambda\}$ indicates the degree of belief that Λ will occur. The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space. In order to obtain an uncertain measure of compound event, a product uncertain measure was defined by Liu [15].

Axiom 4. (Product Axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \ldots$ The product uncertain measure $\mathcal{M}$ is an uncertain measure on the product σ-algebra $\mathcal{L}_1 \times \mathcal{L}_2 \times \cdots$ satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\} \tag{2.2}$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \ldots,$ respectively.

Definition 2.2 (Liu [13]). An uncertain variable is a measurable function from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., $\{\xi \in B\}$ is an event for any Borel set B.