Applied Mathematics and Computation 286 (2016) 265–278

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# Bending of a rectangular plate with rotationally restrained edges under a concentrated force

Wencong Shi[a], X.-F. Li[a,*], C.Y. Wang[b]

[a] School of Civil Engineering, Central South University, Changsha 410075, PR China
[b] Departments of Mathematics and Mechanical Engineering, Michigan State University, East Lansing, MI 48824, USA

ARTICLE INFO

Keywords:
Rectangular plate
Concentrated force
Superposition
Rotationally restrained edge
Corner force

ABSTRACT

The bending problem of a rectangular plate with rotationally restrained edges under a concentrated force is studied. An emphasis is placed on the determination of the corner forces and deflection. The problem is solved by superposition of classical Navier's and Levy's solutions. Analytic solutions in terms of series are obtained for RSSS, RRSS and RSRS plates, respectively, where *R* and *S* stand for rotational restraint and simple support, respectively. Some important design parameters such as maximum deflection and corner force are evaluated. The effect of edge restraint on these parameters is illustrated.



## 1. Introduction

Plates are ubiquitous in engineering applications since they can serve as indispensable components in many structures. Great progress in the research on plates has been made in past decades and the topic still attracts much attention [1]. Prevailing theories and practical applications on plates have been elaborated in some well-known monographs [2,3]. An important class includes rectangular plates which are widely used as gates, windows, slab bridges, ceilings, etc. A variety of studies on different aspects of rectangular plates have been performed [4–7]. Amongst them is the bending of a rectangular plate with various restrained edges and has been comprehensively discussed such as [8,9].

Rectangular plates with restraints against rotation are important since classical boundary conditions including simply-supported and clamped edges can be seen as extreme cases when stiffness of rotational restraints tends to zero and infinity, respectively. Also in practice, the boundary conditions encountered often have supports that mimic a rotationally restrained edge. For such a plate, Johnson [10] analyzed the bending of a plate with four edges supported and one edge subjected to a rotational restraint. Gorman [11] studied the buckling and free vibration of in-plane loaded rectangular plates with opposite rotational edge restraints via superposition of moments proportional to the degree of edge rotation. In another work, Gorman [12] studied the free vibration of a rectangular plate with two symmetrically distributed clamps along one edge, and he superimposed a Levy solution of a plate subject to an edge moment to counterbalance the effect of clamped edge. Li et al. [13] presented a series solution for transverse vibration of rectangular plates with general elastic supports. Other works are concerned about local buckling or free vibration of rotational restrained plates [14–16]. Meleshko reviewed the bending of an elastic rectangular clamped plate in [17]. In particular, for static bending of rectangular plates, Wang [18] recently evaluated the corner forces of at the corners when a rectangular plate is subjected to uniform pressure. It is worth noting that the research on the effect of a concentrated load on a rotationally restrained rectangular plate is still

* Corresponding author. Tel.: +86 73188167070.
E-mail addresses: xfli@csu.edu.cn (X.-F. Li), cywang@mth.msu.edu (C.Y. Wang).

http://dx.doi.org/10.1016/j.amc.2016.04.029

lacking, especially for the corner forces due to an arbitrarily placed load. Furthermore, the concentrated load solution is significant since it can be taken as a Green's function for the solution to other distributed loads.

In this paper, a rectangular plate with arbitrary rotational restraint at four edges is studied. A series solution when the plate subjected to a concentrated load at any position is obtained by superposition. Analytic results are presented for a rectangular plate with one rotationally restrained edge and other three simply-supported edges (RSSS plate), two opposite rotationally restrained edges and other simply-supported edges (RSRS plate), and two adjacent rotationally restrained edged and other two simply-supported edges (RRSS), respectively. An illustrative example is given for a RSSS plate subjected to a point force. Important design parameters such as maximum deflection and corner force are calculated. Numerical results are calculated to presented graphically how the edge restraint affects these parameters.

## 2. Formulation of the problem

Consider a thin plate of length $a$ and width $b$. The plate subjected to a concentrated load at an arbitrary position ($\xi^*$, $\eta^*$), shown in Fig. 1. The edges have zero vertical displacement, but have rotational restraints. When there is no rotational restraint on all edges, the plate is simply supported and can be solved by Navier's method [2]. If the plate has two opposite edges simply-supported, it can be solved by Levy's method [2,3]. However, the problem becomes complicated if all edges have rotational restraints.

Timoshenko and Woinowsky-Krieger [2] considered the case when a plate with two opposite edges simply supported and other two clamped, where the effects of clamped edges can be replaced by rotational moments of inertia. However, there lacks discussions about rotational restraints, and the plate with rotationally restrained edges and subject to concentrated forces were not considered. Gorman [11,12] mainly used superposition principle to address in-plane load and symmetrical edge restraints, thus we have made some broader aims here. To deal with a plate with each edge arbitrarily rotational restrained, we illustrate here a somewhat simpler method by utilizing both Navier and Levy solutions.

Consider a Kirchhoff thin plate of thickness $h$ subjected to transverse loading. Due to no shear deformation, we have two displacement components $u^*$ and $v^*$ along the $x^*$ and $y^*$ directions can be represented by the deflection $w^*$ at the neutral plane, i.e.

$$u^* = -z^* \frac{\partial w^*}{\partial x^*}, \quad v^* = -z^* \frac{\partial w^*}{\partial y^*}. \tag{1}$$

With these assumptions along with the constitutive equations, the internal forces and bending moments of the plate can be expressed in terms of the deflection function $w^*(x^*, y^*)$ at $z = 0$ as follows [2]

$$Q_{x^*} = -D\frac{\partial\left(\nabla^2 w^*\right)}{\partial x^*}, \tag{2}$$

$$Q_{y^*} = -D\frac{\partial\left(\nabla^2 w^*\right)}{\partial y^*}, \tag{3}$$

$$M_{x^*} = -D\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \mu\frac{\partial^2 w^*}{\partial y^{*2}}\right), \tag{4}$$

$$M_{y^*} = -D\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \mu\frac{\partial^2 w^*}{\partial x^{*2}}\right), \tag{5}$$

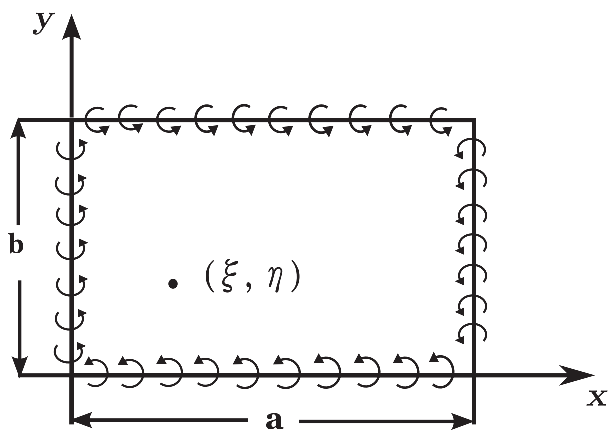


**Fig. 1.** Schematic of a plate subject to a concentrated force with four edges rotational restraints.