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Multiple kinds of optimal impulse control strategies on plant–pest–predator model with eco-epidemiology

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ABSTRACT

Yongzhen et al. (2010) describe a mathematical model of a scenario where a plant population is imported to a pest–predator system with an infected pest. Thus a plant–pest–predator eco-epidemiological model disturbed by an impulsive effect is proposed. First of all, the stability conditions of the susceptible pest-eradication periodic solution for eradicating the susceptible pest are investigated. Compared with the results in (Yongzhen et al., 2010), the presence of the plant population increases the cost of natural enemies as well as the demand for insecticide. In addition, we study the effect of the death rate of the infected pest on pest control in terms of evolution of virulence and the basic reproductive number. Results show that larger mortalities of the infected pest will lead to the frustrated invasion or the instability of susceptible pest-eradication periodic solutions. Next, we focus on the four kinds of optimal impulsive control strategies, biological control, chemical control, and integrated control with fixed period or variable period, to maximize the yields of plants at the terminal time with minimum efforts. All the optimal control problems are solved via a time scaling technique and a gradient-based optimization method. Our results show that two parameters, the amount of sprayed infective pest and the kill fraction of the susceptible pest, play a key role in improving the yield of the plants. In addition, for the four kinds of control strategies, our results also show that biological control is more effective than chemical control to achieve an optimal solution, and the last two strategies can produce higher yields than the first two control strategies.

1. Introduction

Pest control is very important in agriculture because pest outbreak can cause enormous ecological and economic loss. The traditional method is spraying a large variety of chemical pesticides onto the crops. However, pesticide pollution has become a significant problem to human beings and beneficial insects. It has harmful effects on non-target organisms by accumulation of hazardous chemicals in the food chain. Moreover, overuse of chemical pesticides results in the rise of the incidence of pesticide resistance. Therefore, a more effective and safe strategy, integrated pest management (IPM) has been introduced in the late 1950s [1], and more widely developed during the 1970s and 1980s [2,3]. Experience has shown that IPM is more effective than classical methods such as chemical control and biological control [4,5]. Integrated pest management is a systemic and long-term strategy which involves the integration of biological, chemical, and culture tactics

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to reduce pest populations below economically damaging levels [6]. There are several successful examples of the applications of IPM [7,8].

Recently, mathematical models of IPM strategies have been extensively studied. The authors in [9] consider a predator–prey model with disease in the prey. Reference [10] studies a predator–pest model with periodic release of infected pest and predators, which gives sufficient conditions for the stability of periodic solutions and the permanence of the system. The dynamics of a predator–prey food chain with periodic releasing of natural enemies and pesticide spraying is investigated in [11]. In [12], the authors study the stability and permanence of an integrated pest management model with the release of infected pest individuals as well as natural predators, together with the spraying of pesticides.

However, the above references only consider predator–prey models with two species interactions. Few papers explore IPM models described by a three species food web. Gakkhar and Naji [13] study the dynamics of a three-species tri-trophic food web for a generalist-specialist-prey system with modified Holling type II functional response. Further, [14] studies the dynamics of a tri-trophic food web system containing a Leslie–Gower type generalist predator. Based on a two species predator–prey model [12], this paper proposes a three species model which includes a top predator(the enemy of the pest), the pest, and a basal producer (a renewable biotic resource, referred to as plant below). The pest is the prey of the predator, and the generalist predator of the basal producer. Compared to the model in [12], we show that the presence of basal producer has a destabilizing effect in the sense that solutions exhibit oscillations instead of approaching the susceptible pest-eradication periodic solution in its absence.

Pest control is required for improving economic yields. Hence, the goal of this paper is to maximize the economic yield with minimum cost. Two commonly used tactics, such as chemical control and biology control, are applied to realize the goal. That is, spraying pesticides or releasing an infectious pest population and predator are used to control the number of the pest. These two impulsive controls are implemented at fixed periodic time points. In addition, we also consider implementation of the impulsive controls at variable impulse time points. That is, the impulse time points are chosen as decision variables. Intuitively, variable time impulse controls can obtain higher yield than the fixed time case. It is also effective and practical [15]. For example, in state dependent impulse control, instead of implementing impulse controls at fixed moments, the measures for pest control are only taken when the amount of pest reaches a threshold value. In this case, the impulse instants are varying in the time horizon.

Optimal control theory of impulsive systems, and both analytical and numerical techniques are now well developed [16–22]. In [16], an impulsive control problem is treated as a discrete-continuous (hybrid) model, and then the gradients of the cost functional with respect to time moments of impulses can be obtained. Based on the parameterization of the switching instants, an optimal control problem for switched systems is transformed into an equivalent optimization problem and then the derivatives of the cost function with respect to the switching time are obtained based on the solution of a two point boundary value differential algebraic equation [17]. The authors in [18] present an approach for computing the derivative of the optimal value function based on analyzing the differentiability of the cost function. Based on a time scaling transformation, the method proposed in [19–21] has been widely used to optimize impulse times, and its advantages are rigorously explained in [22]. Hence, this paper applies the latter method to obtain the gradients of the objective function with respect to the control variables so that gradient-based optimization method can be used to find the optimal solutions.

The paper is organized as follows. Section 2 gives the new three species mathematical model and the main biological assumptions. The next section analyzes the dynamical properties of the new model. It gives the sufficient conditions for local and global stability of susceptible pest-eradication periodic solutions. Section 4 focuses on finding different optimal control strategies to maximize the economic yields at the terminal time via a gradient-based optimization method. The gradients of objective function with respect to controls are also given in this section. Finally a brief discussion of the results is presented.

2. Model formulation

On the basis of the predator–prey model proposed in [12], a new IPM model is formulated involving a food for the pest. At time $t \neq nT,\ \ n = 1, 2, \ldots, N$, the three species plant, pest and predator satisfy the following ordinary differential equations:

$$\begin{cases} \dot{S} = rS(1 - \dfrac{S+I}{K}) - \lambda IS - \dfrac{\alpha_2 SP}{1+a_2 S} + \dfrac{\beta_1 SX}{1+a_1 X} \\ \dot{I} = \lambda IS - \mu I \\ \dot{P} = \dfrac{\beta_2 SP}{1+a_2 S} - \gamma P \\ \dot{X} = bX(1 - \dfrac{X}{L}) - \dfrac{\alpha_1 SX}{1+a_1 X} \end{cases} \tag{1}$$