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Reachability of higher-order logical control networks via matrix method[☆]

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ABSTRACT

This paper investigates the reachability of higher-order logical control networks. First, with semi-tensor product method, the matrix expression of a higher-order logical control network is given. Then, a partitioned matrix is constructed, which intuitively shows the input-state mapping information. With this matrix, some conditions are obtained for the reachability of higher-order logical control networks. Finally, an algorithm is designed to find control sequences that drive a given initial state to a given destination state.

1. Introduction

A Boolean network (BN) is a 2-valued logical network, and each of its nodes takes value 0 or 1. It was first proposed by Kauffman [1] to describe, analyze and simulate genetic regulatory networks. In a genetic regulatory network, a gene state is quantized to True $\sim$ 1 or False $\sim$ 0, and the state of each gene is determined by the states of its neighborhood genes using logical rules. In recent decades, BNs have received considerable attention from the community of systems biology, e.g., Shmulevich et al. [2] discussed the relationship of probabilistic BNs and Bayesian networks, Chaves et al. [3] and Albert et al. [4] used Boolean modeling framework to study the segment polarity genes, and Akutsu et al. [5] used BNs to analyze the identifying problem of a genetic network.

BNs with external inputs are usually called Boolean control networks (BCNs). Since the concept of BCN was proposed, it has been used to study many issues, e.g., Pal et al. [6] extended the method of external control to context-sensitive probabilistic BNs, Datta et al. [7] presented that a control strategy can be implemented in the imperfect information case, and Fauré et al. [8] analyzed the dynamical properties of a Boolean control model.

A logical control network (LCN) has a structure similar to that of a BCN, but each of its nodes takes value from a finite set, e.g., each node of a k-valued LCN takes value from $\{0, \frac{1}{k-1}, \dots, \frac{k-2}{k-1}, 1\}$. Particularly, a BCN is a 2-valued LCN. When the updated values of a LCN depend on the past μ values and the past μ inputs, it is called a μth order LCN. This model was first proposed in [9] to solve the optimal control problem, and has been proved to be very useful in game theory.

Recently, the semi-tensor product method proposed by Cheng [10] has been used to analyze LCNs. It is a new technique that can convert the logical dynamic equations of LCNs into discrete-time dynamic equations. With this new technique, LCNs have rapidly attracted substantial research interests in the community of control theory. Many classical control problems have been extended to LCNs, e.g., topology structure [11–13], controllability and observability [14–19], stability and

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state feedback stabilization [20–23], system decomposition [24,25], state observers [26], disturbance decoupling [27,28], realization [29], and optimal control [30–32].

Reachability is one of the fundamental concepts in control theory, and many results have appeared on the reachability of BCNs. Cheng and Qi [14] proposed the concept of reachability and Zhao et al. [15] solved the reachability problem of BCNs with input-state incidence matrix method. Based on the results in [14,15], Li and Wang [16] analyzed the reachability of switched BCNs, Liu et al. [17] discussed the reachability of probabilistic BCNs and Li and Sun [18] studied the reachability of higher-order BCNs. From the reachability of a system, we can determine whether there exists a control sequence that steers the system from an undesirable location to a desirable one. Moreover, reachability is the fundamental content to solve other problems like observability, stability and stabilization, observer design and optimal control.

This paper investigates the reachability of higher-order LCNs. It is worth noting that Li and Sun [18] studied the reachability of higher-order BCNs. Compared with a μth order BCN in [18], a μth order LCN has its updated values depending on not only the past μ values but also the past μ inputs instead of the current inputs. The dependence on the past μ inputs makes the calculation more complex, and there does not exist an efficient tool to deal with the past μ inputs. Thus, the reachability of higher-order LCNs is more difficult.

Zhao and Cheng [19] solved the reachability problem of higher-order LCNs by introducing the input-state incidence matrices of restricted deterministic LCNs. Since it is difficult to construct the input-state incidence matrix of a restricted deterministic LCN, it is still a challenging issue to study the reachability of higher-order LCNs. Here, we give a new matrix method to investigate the reachability of higher-order LCNs. This method can intuitively show the input-state mapping information and easily give the reachable set of higher-order LCNs.

This paper is organized as follows. Section 2 provides some preliminaries. Section 3 states the reachability problem of higher-order LCNs. To solve the reachability problem, a μth order LCN is transformed into an alternative form. Section 4 analyzes the reachability of higher-order LCNs by a matrix method. An algorithm is designed in Section 5. Section 6 is a brief conclusion.

2. Preliminaries

Notations. Throughout the whole paper, we use the following notations.

1. Denote the set of all the $m \times n$ real matrices by $\mathbb{R}_{m\times n}$.
2. Let Col(A) be the set of all the columns of matrix A. Denote the ith column of A by Col$_i(A)$.
3. Set $\Delta_k = \{\delta_k^i | i = 1, 2, \dots, k\}$, where δ_k^i is the ith column of identity matrix I_k. For simplicity, we denote $\Delta := \Delta_2 = \{\delta_2^1, \delta_2^2\}$.
4. A matrix $L \in \mathbb{R}_{m\times n}$ is called a logical matrix if Col(L)$\subset \Delta_m$. Denote the set of all the $m \times n$ logical matrices by $\mathcal{L}_{m\times n}$. For simplicity, we denote the logical matrix $L = [\delta_m^{i_1}\delta_m^{i_2}\cdots\delta_m^{i_n}]$ by $L = \delta_m[i_1 i_2 \cdots i_n]$.
5. Let $\mathbf{1}_n$ be the n-dimensional column vector whose entries are all equal to 1.
6. Set $\mathcal{D}_k = \{0, \frac{1}{k-1}, \dots, \frac{k-2}{k-1}, 1\}$ and $\mathcal{D} := \mathcal{D}_2 = \{0, 1\}$.
7. Define the m-dimension power-reducing matrix by $M_{r,m} := [\delta_m^1 \otimes \delta_m^1 \delta_m^2 \otimes \delta_m^2 \cdots \delta_m^m \otimes \delta_m^m]$. It satisfies $X^2 = M_{r,m}X$, $X \in \Delta_m$.
8. Let $A \in \mathbb{R}_{m\times mn}$. Denote the ith $m \times m$ square block of A by Blk$_i(A)$, $i = 1, 2, \dots, n$.

Set $A \in \mathbb{R}_{m\times n}$, $B \in \mathbb{R}_{p\times q}$ and $\alpha = \mathrm{lcm}(n, p)$, i.e. the least common multiple of n and p. The tensor (Kronecker) product [33] of $A = (a_{ij})$ and B is defined as

$$A \otimes B = (a_{ij}B).$$

The semi-tensor product [11] of A and B is defined as

$$A \ltimes B = (A \otimes I_{\frac{\alpha}{n}})(B \otimes I_{\frac{\alpha}{p}}).$$

For the basic properties of the semi-tensor product, please refer to [10,11]. Since the main properties of the traditional matrix product remain true for the semi-tensor product, the semi-tensor product is a generalization of the traditional matrix product. In this paper, the symbol $\ltimes$ is omitted, i.e. $A \ltimes B$ is directly written as AB.

Let matrices A, B, C and D have proper dimensions. A useful property [33] of the tensor product is

$$AC \otimes BD = (A \otimes B)(C \otimes D). \tag{1}$$

Let $X \in \Delta_m$, $Y \in \Delta_n$. Then $\mathbf{1}_m^{\mathrm{T}}X = 1$, $\mathbf{1}_n^{\mathrm{T}}Y = 1$ and

$$XY = X \otimes Y, \tag{2}$$

$$X = (I_m X) \otimes (\mathbf{1}_n^{\mathrm{T}}Y) = (I_m \otimes \mathbf{1}_n^{\mathrm{T}})XY, \tag{3}$$

$$Y = (\mathbf{1}_m^{\mathrm{T}}X) \otimes (I_n Y) = (\mathbf{1}_m^{\mathrm{T}} \otimes I_n)XY. \tag{4}$$