Applied Mathematics and Computation 287–288 (2016) 83–93

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Numerical solutions of hyperbolic telegraph equation by using the Bessel functions of first kind and residual correction

Şuayip Yüzbaşı*

Department of Mathematics, Faculty of Science, Akdeniz University, TR 07058 Antalya, Turkey

A R T I C L E I N F O

Keywords:
Hyperbolic telegraph equation
Bessel functions of first kind
Bessel collocation method
Collocation points
Residual function
Residual correction

A B S T R A C T

In this study, a collocation method is presented to solve one-dimensional hyperbolic telegraph equation. The problem is given by hyperbolic telegraph equation under initial and boundary conditions. The method is based on the Bessel functions of the first kind. Using the collocation points and the operational matrices of derivatives, we reduce the problem to a set of linear algebraic equations. The determined coefficients from this system give the coefficients of the approximate solution. Also, an error estimation method is presented for the considered problem and the method. By using the residual function and the original problem, an error problem is constructed and thus the error function is estimated. By aid of the estimated function, the approximated solution is improved. Numerical examples are given to demonstrate the validity and applicability of the proposed method and also, the comparisons are made with the known results.

1. Introduction

The linear or nonlinear partial differential equations have an important place in science and engineering because they are encountered in mathematical modeling. The vibrations of structures (e.g. buildings, beams and machines) are modeled by the hyperbolic partial differential equations. Also, the telegraph equation is mostly used in wave propagation of electric signals in a cable transmission line [1]. Recently, the hyperbolic telegraph equation have been solved by the numerical methods such as collocation method based on splines radial basis function [2], the Chebyshev Tau method [3], Legendre multi-wavelet Galerkin method [4], Homotopy perturbation method [5], variational iteration and homotopy perturbation methods [6], the finite difference scheme [7], the DGJ Method [8], Chebyshev spectral collocation method [9], the differential quadrature method [10], the Haar wavelet method [11], the dual reciprocity boundary integral equation method [12] and the variational iteration method [13].

In recent years, various matrix and collocation methods and the polynomial approximations [2,9, 14–25] have been used for solutions of differential equations and integral equations.

In this paper, we consider a numerical approximation for solving the second-order hyperbolic problem given in [2–6]:

$$\frac{\partial^2 u}{\partial t^2} + \alpha \frac{\partial u}{\partial t} + \beta u = \frac{\partial^2 u}{\partial x^2} + F(x,t), \quad 0 \le x \le L, \quad 0 \le t \le T, \tag{1}$$

with the initial conditions

$$u(x,0) = g_0(x), \quad u_t(x,0) = g_1(x), \quad 0 \le x \le L, \tag{2}$$

* Tel.: + 90 242 310 23 83; fax: + 90 242 227 89 11.
E-mail address: syuzbasi@akdeniz.edu.tr, suayipyuzbasi@gmail.com

http://dx.doi.org/10.1016/j.amc.2016.04.036
0096-3003/

and the boundary conditions

$$u(0,t)=h_0(t),\quad u(L,t)=h_1(t),\quad 0\le t\le T, \tag{3}$$

where α, β are given constant coefficients, $F(x,t)$, $g_0(t)$, $g_1(t)$, $h_0(t)$, $h_1(t)$ are known functions and $u(x,t)$ is the unknown function. For hyperbolic telegraph problems (1) and (2) in this paper, we will find the approximate solutions expressed in the truncated Bessel series form

$$u(x,t)=\sum_{r=0}^{N}\sum_{s=0}^{N}a_{r,s}J_{r,s}(x,t);\quad J_{r,s}(x,t)=J_r(x)J_s(t), \tag{4}$$

where $a_{r,s}$; $r,s=0,\dots,N$ are the unknown Bessel coefficients and $J_n(x)$, $n=0,1,2,\dots,N$ are the Bessel functions of first kind defined by

$$J_n(x)=\sum_{k=0}^{[[\frac{N-n}{2}]]}\frac{(-1)^k}{k!(k+n)!}\left(\frac{x}{2}\right)^{2k+n},\quad n\in\mathbf{N},\quad 0\le x<\infty.$$

2. Required matrix relations for solution method

To obtain the numerical solution of the hyperbolic telegraph equation with the presented method, we will evaluate the Bessel coefficients of the unknown function. For convenience, the solution function (4) can be written in the matrix form

$$u(x,t)=\mathbf{J}(x)\mathbf{Q}(t)\mathbf{A}, \tag{5}$$

where

$$\mathbf{J}(x)=[J_0(x)\;J_1(x)\cdots J_N(x)]_{1\times(N+1)},\quad \mathbf{Q}(t)=\begin{bmatrix}\mathbf{J}(t)&0&\cdots&0\\0&\mathbf{J}(t)&\cdots&0\\0&0&\ddots&\vdots\\0&0&0&\mathbf{J}(t)\end{bmatrix}_{(N+1)\times(N+1)^2}$$

$$\mathbf{A}=[a_{0,0}\;a_{0,1}\cdots a_{0,N}\;a_{1,0}\;a_{1,1}\cdots a_{1,N}\cdots a_{N,0}\;a_{N,1}\cdots a_{N,N}]^T,$$

$$\mathbf{J}(x)=\mathbf{X}(x)\mathbf{D}^T,\quad \mathbf{X}(x)=[1\;x\;x^2\cdots x^N] \tag{6}$$

and the coefficient matrix $\mathbf{D}$:
if N is odd,

$$\mathbf{D}=\begin{bmatrix}\frac{1}{0!0!2^0}&0&\frac{-1}{1!1!2^2}&\cdots&\frac{(-1)^{\frac{N-1}{2}}}{(\frac{N-1}{2})!(\frac{N-1}{2})!2^{N-1}}&0\\0&\frac{1}{0!1!2^1}&0&\cdots&0&\frac{(-1)^{\frac{N-1}{2}}}{(\frac{N-1}{2})!(\frac{N+1}{2})!2^N}\\0&0&\frac{1}{0!2!2^2}&\cdots&\frac{(-1)^{\frac{N-3}{2}}}{(\frac{N-3}{2})!(\frac{N+1}{2})!2^{N-1}}&0\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&\cdots&\frac{1}{0!(N-1)!2^{N-1}}&0\\0&0&0&\cdots&0&\frac{1}{0!N!2^N}\end{bmatrix}_{(N+1)\times(N+1)},$$

if N is even,

$$\mathbf{D}=\begin{bmatrix}\frac{1}{0!0!2^0}&0&\frac{-1}{1!1!2^2}&\cdots&0&\frac{(-1)^{\frac{N}{2}}}{(\frac{N}{2})!(\frac{N}{2})!2^N}\\0&\frac{1}{0!1!2^1}&0&\cdots&\frac{(-1)^{\frac{N-2}{2}}}{(\frac{N-2}{2})!(\frac{N}{2})!2^{N-1}}&0\\0&0&\frac{1}{0!2!2^2}&\cdots&0&\frac{(-1)^{\frac{N-2}{2}}}{(\frac{N-2}{2})!(\frac{N+2}{2})!2^N}\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&\cdots&\frac{1}{0!(N-1)!2^{N-1}}&0\\0&0&0&\cdots&0&\frac{1}{0!N!2^N}\end{bmatrix}_{(N+1)\times(N+1)}.$$

On the other hand, the relation between the matrix $\mathbf{X}(x)$ and its k-th order derivative $\mathbf{X}^{(k)}(x)$ is

$$\mathbf{X}^{(k)}(x)=\mathbf{X}(x)(\mathbf{B}^T)^k, \tag{7}$$