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A note on exponential stability for second-order neutral stochastic partial differential equations with infinite delays in the presence of impulses[☆]

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ABSTRACT

In this work, the problem on the exponential stability for second-order neutral stochastic partial differential equations with infinite delays is considered in the presence of impulses under some conditions. By employing the new integral inequality technique, some algebraic criteria of stability are established for the concerned problem and some existing results are generalized and improved. Finally, an illustrative example is given to demonstrate the effectiveness of the obtained results.

1. Introduction

Over the last decades, stability of solutions for stochastic partial differential equations in abstract spaces has been discussed extensively in many areas such as engineering, biological systems and physical phenomena [1,2].

As is known to all, Itô formula in infinite dimensions cannot be used directly for mild solutions of stochastic partial differential equations due to their non-differentiability. However, fortunately, the methods of studying mild solutions of stochastic partial differential equations have been developed by the fixed point theory and some integral inequalities [3–6]. The authors in [7,8] discussed stochastic evolutions equations driven by fractional Brownian motion. Revathi et al. [9] studied the existence of almost automorphic mild solutions of neutral stochastic differential equations. In addition, we know that impulsive noises often occur in the practice and they can make nonlinear systems change abruptly and disperse stability of systems. Now the study on stability of mild solutions of some impulsive stochastic partial differential equations has attracted lots of attention [10–12]. For example, Ren et al. [13] studied impulsive neutral stochastic functional integro-differential equations with infinite delay. Arthi [14] discussed stability of impulsive neutral stochastic fractional integrodifferential equations.

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Recently, stability and controllability of mild solutions of second-order stochastic partial differential equations have become popular due to their applications in modeling mechanical vibrations. The focus on second-order equations is to study them directly rather than make them become first-order equations [15]. For example, the authors in [16,17] studied asymptotic stability and approximate controllability for second-order stochastic evolution equations. Arthi et al. [18,19] studied controllability of second-order impulsive stochastic evolution systems and second-order impulsive neutral integrodifferential systems, respectively. Base on the integral inequalities, Arthi et al. [20] discussed exponential stability of second-order impulses neutral stochastic differential equations with constant delays. By employing integral inequalities, Chen [21] investigated the exponential stability and asymptotic stability for second-order neutral stochastic partial differential equations with infinite delay. Sakthivel and Ren [22,23] generalized second-order stochastic evolution equations to more general Poisson jumps cases. In [24], the authors investigated controllability of second-order neutral stochastic differential equations with infinite delay and Poisson jumps. In order to overcome the difficulty of impulses and infinite delays, in this paper we study second-order neutral stochastic partial differential equations with infinite delays in the presence of impulses by establishing a new integral inequality.

In this note, we consider the following second-order neutral stochastic partial differential equations with infinite delays in the presence of impulses:

$$\begin{cases} d[z'(t)-\mu(t,z_t)]=[Az(t)+f(t,z_t)]dt+g(t,z_t)dB(t), t\geq 0, t\neq\tau_j, j=1,2,\dots, \\ \Delta z(\tau_j)=I_j(z(\tau_j^-))), j=1,2,\dots, \\ \Delta z'(\tau_j)=\hat{I}_j(z(\tau_j^-))), j=1,2,\dots, \\ z_0(\cdot)=\varphi\in\mathfrak{C}, \quad z'(0)=\xi, \end{cases} \tag{1}$$

where ξ is an $\mathfrak{F}_0$-measurable $\mathcal{Z}$-valued random variable independent of the Wiener process $B(t)$. $\mu, f: [0,+\infty)\times\mathfrak{C}\to\mathcal{Z}$, $g: [0,+\infty)\times\mathfrak{C}\to\mathfrak{L}_2^0(\Upsilon,\mathcal{Z})$.$A: \mathfrak{D}(A)\subset\mathcal{Z}\to\mathcal{Z}$ is the infinitesimal generator of a strongly continuous cosine family on $\mathcal{Z}$. Let $z_t: (-\infty,0]\to\mathcal{Z}$ and $z_t(\vartheta)=z(t+\vartheta)$ $(t\geq 0)$ be in the space $\mathfrak{C}$. $0<\tau_1<\tau_2<\dots<\tau_j<\cdots$ and $\lim_{j\to+\infty}\tau_j=+\infty$. $I_j,\hat{I}_j: \mathfrak{C}\to H$, $\Delta\xi(t)=\xi(t^+)-\xi(t^-)$, where $\xi(t^-)$ and $\xi(t^+)$ denote the left and right limit of ξ at t, respectively.

In this paper, by establishing a new integral inequality with impulses and infinite delays, we will give some algebraic criteria of pth exponential stability of second-order neutral stochastic partial differential equations with infinite delays in the presence of impulses. The rest of this note is organized as follows. In Section 2, some preliminaries are introduced and then in Section 3 stability of second-order neutral stochastic partial differential equations with infinite delays in the presence of impulses is given under some weaker conditions by establishing a new integral inequality. Finally, we give an example to show the theoretical results.

2. Preliminaries

Throughout this note, let $(\Omega,\mathfrak{F},\{\mathfrak{F}_t\}_{t\geq 0},\mathcal{P})$ be a complete probability space with the filtration $\mathfrak{F}_t(t\geq 0)$ satisfying the usual conditions. Let $\mathcal{Z}$ and Y be two real, separable Hilbert spaces and $\mathfrak{L}(\Upsilon,\mathcal{Z})$ be the space of bounded linear operators from Y to $\mathcal{Z}$. Let $\|\cdot\|$ be the norms in $\mathcal{Z}$, Υ and $\mathfrak{L}(\Upsilon,\mathcal{Z})$. Let $C((-\infty,0],\mathcal{Z})$ be the space of all bounded and continuous functions φ from $(-\infty,0]$ to $\mathcal{Z}$ with the norm $\|\cdot\|_C=\sup_{-\infty<\vartheta\leq 0}\|\varphi(\vartheta)\|$ and the space $\mathfrak{C}$ be the family of all $\mathfrak{F}_t(t\geq 0)$-measurable and $C((-\infty,0],\mathcal{Z})$-valued random variables.

Let $Q\in\mathfrak{L}(\Upsilon,\Upsilon)$ and the Y-valued stochastic process $B(t)$ be a Q-Wiener process. σ is a Q-Hilbert–Schmidt operator with $\|\sigma\|^2_{\mathfrak{L}_2^0}:=\mathrm{tr}(\sigma Q\sigma^*)<+\infty$, where $\mathfrak{L}_2^0(\Upsilon,\mathcal{Z})$ denotes the space of all Q-Hilbert–Schmidt operators $\sigma: \Upsilon\to\mathcal{Z}$. For the details, the readers can refer to [1]. Let $\{C(t): t\in R\}\subset\mathfrak{L}(\mathcal{Z},\mathcal{Z})$ be a strongly continuous cosine family and $\{S(t): t\in R\}\subset\mathfrak{L}(\mathcal{Z},\mathcal{Z})\}$ be a strongly continuous sine family with $S(t)z=\int_0^t C(s)zds$, $t\in R$, $z\in\mathcal{Z}$ (see [21,23]). The generator $A: \mathcal{Z}\to\mathcal{Z}$ of $\{C(t): t\in R\}$ is given by $Az=\frac{d^2}{dt^2}C(t)z|_{t=0}$ for all $z\in\mathfrak{D}(A)=\{z\in\mathcal{Z}: C(\cdot)z\in C^2(R,\mathcal{Z})\}$.

Lemma 2.1 [1]. *For any $r\geq 1$ and $\mathfrak{L}_2^0(\Upsilon,\mathcal{Z})$-valued predictable process $\phi(\cdot)$ such that*

$$\sup_{s\in[0,t]}E\|\int_0^s\phi(u)dB(u)\|^{2r}\leq C_r\Big(\int_0^t(E\|\phi(s)\|^{2r}_{\mathfrak{L}_2^0})^{\frac{1}{r}}ds\Big)^r, \quad t\in[0,+\infty),$$

where $C_r=(r(2r-1))^r$.

Definition 2.1. An $\mathcal{Z}$-value stochastic process $z(t)$, $t\in R$, is called a mild solution of Eq. (1), if

(i) $z(t)$ is $\mathfrak{F}_t$-adapted and has càdlàg path on $t\geq 0$ almost surely;
(ii) for $t\in[0,+\infty)$, almost surely

$$z(t)=C(t)\varphi+S(t)(\xi-\mu(0,z_0))+\int_0^t C(t-r)\mu(r,z_r)dr$$
$$+\int_0^t S(t-r)f(r,z_r)dr+\int_0^t S(t-r)g(r,z_r)dB(r)$$