Applied Mathematics and Computation 285 (2016) 8–25

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

An enhanced exponential matrix approach aimed at the stability of piecewise beams on elastic foundation

Eugenio Ruocco[a,*], Vincenzo Mallardo[b]

[a] Second University of Naples, Deaprtment of Civil Engineering, Design, Building and Environment, Via Roma 28, 81031 Aversa, Caserta, Italy

[b] University of Ferrara, Department of Architecture, Via della Ghiara 36, 44121 Ferrara, Italy

ARTICLE INFO

Keywords:
Buckling
Exponential Matrix Method
Computational methods

ABSTRACT

In this paper, an enhanced exponential collocation method for the stability assessment of piecewise beams resting on elastic foundation and subjected to both non-uniform distributed and concentrated loads is presented. The exponential basis functions usually adopted in literature are enriched with trigonometric functions that make the eigenvalue analysis well-posed without reducing the original simplicity of the method. A measure of the error occurring from the proposed approach and its improvement are also proposed. Several examples aimed at estimating the buckling loads under typical end supports are discussed. A comparison with exact and with other numerical results that are available in literature are carried out. Such a comparison shows the high accuracy and the fast convergence of the proposed approach.

1. Introduction

Buckling analysis of thin-walled structures subjected to the simultaneous action of combined loading has strong relevance in structural, mechanical and aeronautical engineering fields [1]. Starting with the book of Gerard and Backer [2], a great amount of literature has been published on the stability of structures under combined beams. The analytical solutions are extremely rare: Duan and Wang [3] derived the analytical solutions for the elastic buckling of heavy columns in terms of generalized hypergeometric functions. Wang et al. [4] gave the exact critical load of a wide range of structural members, including columns, arches, rings, plates and shells under simple load conditions. In [5,6] the authors proposed a closed form solution for measuring the effect of the non linear strain terms usually neglected under the von Kàrmàn hypothesis on the flexo-torsional buckling of beams, plates and shells.

As analytical solutions are available only in simple cases, various numerical methods have been proposed in literature for solving the buckling problems of beam and columns with general geometries and load conditions. A numerical method based on the differential quadrature method for the buckling and post-buckling analysis of extensible beam–columns was proposed by Yuan and Wang [7] and used for solving six different Euler buckling cases. Civalek and Yavas [8] used the discrete singular convolution method to analyse buckling problems of columns having different geometries. Wang and Ang [9] obtained the buckling behavior of heavy columns by adopting a trigonometric series for approximating the deflection curve and by solving the quadratic characteristic equation supplied through the Rayleigh-Ritz energy approach.

* Corresponding author. Tel.: +390815010266; Fax: +39 081 5037370.
E-mail address: eugenio.ruocco@unina2.it, euruocco@gmail.com (E. Ruocco).

http://dx.doi.org/10.1016/j.amc.2016.03.020
0096-3003/

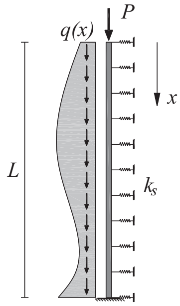

Fig. 1. Example of a loaded column resting on an elastic Winkler type foundation.

On the other hand, foundation beams as well as retaining walls, both dealt with as structures on elastic foundation, have wide applications in modern engineering and pose great technical problems in structural design. Particularly, the modern approach aimed at designing more and more slender structural elements poses new stability issues. The Winkler elastic foundation beam model, which consists of a beam over an infinitely close regularly spaced linear springs, is a one-parameter model that has been extensively used in practice. A variational interation method for solving the stability problem of Euler columns restrained along its length was proposed by Atay and Coşkun [10]. A new and simple approach was presented by Huang and Luo [11] to solve buckling of axially inhomogeneous beams with a continuous elastic restraint. By using the power series they obtained a characteristic equation of the critical buckling loads in a polynomial form, whose corresponding root can be easily obtained. No variation of either axial load or beam section is included in the formulation.

In this paper, an Exponential Matrix Method (EMM) [12] for the buckling analysis of piecewise columns resting on elastic foundation and subjected to both distributed and concentrated non-uniform loads is presented. The EMM has been successfully adopted in many branches of mechanics, conceivably because both its simplicity makes it applicable in a systematic way, and it can be easily implemented in a hand-made program code. For instance, Çevik et al. [13] solved a delayed single degree of freedom system equation by EMM. They applied the method for finding the solution of the equations of a linear oscillator with delay. More recently, Yüzbaşı and Sezer [14] presented a generalized version of the method aimed at solving systems of high-order linear differential equations with variable coefficients. By using the Tau method [15] and the residual correction method [16], the authors in [17] estimated an error measure of the proposed approximate solution, and he improved it until a required accuracy is achieved.

It must be pointed out that to the authors' knowledge no applications of the EMM to structural stability are found in the literature.

However, the EMM so far developed in literature presents some drawbacks which make its applicability to the buckling analysis of a piecewise beam computationally inefficient. The usual approach involves exponential functions that do not suit properly to the buckling load analysis. In fact, the corresponding eigenvalue problem may become ill-conditioned. The drawback is amplified with increasing number of terms of the exponential series.

In the present paper an enhanced approach is proposed to overcome the underlined problems: (i) two orthogonal trigonometric-type functions are added to the common exponential basis, in order to improve the kinematics of the model without influencing the inherent advantages related to the use of exponential functions, (ii) the a-coefficients, usually adopted as unknown in the classical EMM, are replaced by some nodal displacements that govern the out-of-plane behavior of an Euler beam. On the basis of the above improvements the buckling analysis results to be governed by an approximate stiffness matrix of dimension 4×4, independently on the number of basis functions adopted. Moreover, the intrinsic properties of the stiffness matrix (symmetry, if unbuckled, positive definiteness with positive diagonal elements) make the numerical procedure more efficient. Finally, the introduction of the stiffness matrix allows the employment of a finite element method procedure, and, thus, the analysis of structures with more complex geometries and/or boundary conditions, by simply subdividing the beam in sub-elements.

2. The basic equation

Consider an elastic column of length L and stiffener EI, embedded in a Winkler elastic medium with the spring constant k_s, and subjected to an axial compressive force P and/or to a distributed axial loading $q(x)$ defined in terms of the axial coordinate x, measured from the bottom end (Fig. 1). Assuming the Euler–Bernouilli kinematical hypothesis, the governing differential equation is given by:

$$EI(EI * d^4w/dx^4) + \frac{d^4w}{dx^4} + P\frac{d^2w}{dx} + \frac{d}{dx}\left(\int q(x)\frac{dw}{dx}\right) + k_s w = 0 \tag{1}$$

that is:

$$w^{(4)} + (\alpha + \beta(x))w^{(2)} + \beta^{(1)}(x)w^{(1)} + cw = 0 \tag{2}$$