Applied Mathematics and Computation 285 (2016) 94–102

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Some subclasses of close-to-convex mappings associated with conic regions

H.M. Srivastava[a,b,*], Muhammad Rafiullah Khan[c], Muhammad Arif[c]

[a] Department of Mathematics and Statistics, University of Victoria, Victoria, British Columbia V8W 3R4, Canada
[b] Center for General Education (Department of Science and Technology), China Medical University, Taichung 40402, Taiwan, Republic of China
[c] Department of Mathematics, Abdul Wali Khan University, Mardan 23200, Pakistan

ARTICLE INFO

MSC:
Primary 30C45
Secondary 30C50,

Keywords:
Analytic and univalent functions
Principle of subordination
Conic domain
Uniformly close-to-convex functions
Coefficient estimates and arc-length problem
Radius of convexity

ABSTRACT

In this paper, we introduce and investigate several new subclasses of close-to-convex functions and their related mappings. Various interesting properties including sufficiency criteria, coefficient estimates, arc-length problem and radius of convexity are investigated.

1. Introduction and definitions

Let $\mathcal{A}$ be the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

We denote by $\mathcal{S}$ the class of all functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$. Suppose also that, for $0 \leqq \beta < 1$, $\mathcal{S}^*(\beta)$, $\mathcal{C}(\beta)$ and $\mathcal{K}(\beta)$ denote, respectively, the classes of functions in $\mathcal{A}$ which are starlike, convex and close-to-convex of order β in $\mathbb{U}$ (see, for details, [2]).

If f and g are analytic in $\mathbb{U}$, we say that f is subordinate to g, written as

$$f \prec g \quad \text{or} \quad f(z) \prec g(z),$$

if there exists a Schwarz function w, which is analytic in $\mathbb{U}$ with

$$w(0) = 0 \quad \text{and} \quad |w(z)| < 1,$$

* Corresponding author at: Department of Mathematics and Statistics, University of Victoria, Victoria, British Columbia V8W 3R4, Canada. Tel.: +1 2504725313.
E-mail addresses: harimsri@math.uvic.ca (H.M. Srivastava), rafilec@gmail.com (M. Rafiullah Khan), marifmaths@yahoo.com (M. Arif).

http://dx.doi.org/10.1016/j.amc.2016.03.025

such that

$$f(z) = g\big(w(z)\big) \qquad (z \in \mathbb{U}).$$

Furthermore, if the function $g(z)$ is univalent in $\mathbb{U}$, then the following equivalence holds true (see [2,8]):

$$f(z) \prec g(z) \qquad (z \in \mathbb{U}) \iff f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U})$$

Let

$$\Omega_k = \left\{ u + iv : u > k\sqrt{(u-1)^2 + v^2} \qquad (u, v \in \mathbb{R}) \right\}. \tag{1.2}$$

Then, for a fixed value of k, the domain Ω_k represents the conic region bounded by an ellipse for $k > 1$, the right half-plane when $k = 0$, the right branch of a hyperbola whenever $0 \leqq k < 1$ and the region Ω_k is parabolic for the value $k = 1$. The domain Ω_k was studied by Kanas *et al.* (see, for details, [3–6]). Here, in our present investigation, we define the domain $\Omega_{k,\gamma}$, related to the domain Ω_k given by (1.2), as follows (see [10]):

$$\Omega_{k,\gamma} = \gamma\Omega_k + (1-\gamma) \qquad \big(0 < \Re(\gamma) \leqq k+1\big).$$

Extremal functions for these conic regions, denoted by $p_{k,\gamma}(z)$, are analytic in $\mathbb{U}$ and map the open unit disk $\mathbb{U}$ onto the domain $\Omega_{k,\gamma}$ such that

$$p_{k,\gamma}(0) = 1 \qquad \text{and} \qquad p'_{k,\gamma}(0) > 1,$$

where $p_{k,\gamma}(z)$ is given by

$$p_{k,\gamma}(z) = \begin{cases} \dfrac{1+(2\gamma-1)z}{1-z} & (k=0) \\ 1 + \dfrac{2\gamma}{\pi^2}\left[\log\left(\dfrac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right]^2 & (k=1) \\ 1 + \dfrac{2\gamma}{1-k^2}\sinh^2\left[\left(\dfrac{2}{\pi}\arccos k\right)\operatorname{arctanh}(\sqrt{z})\right] & (0<k<1) \\ 1 + \dfrac{\gamma}{k^2-1}\sin\left(\dfrac{\pi}{2R(t)}\displaystyle\int_0^{\frac{u(z)}{\sqrt{t}}}\dfrac{1}{\sqrt{1-x^2}\sqrt{1-(tx)^2}}\,dx\right) + \dfrac{1}{k^2-1} & (k>1), \end{cases} \tag{1.3}$$

where

$$u(z) = \frac{z-\sqrt{t}}{1-\sqrt{tz}} \qquad (0 < t < 1;\ z \in \mathbb{U})$$

such that

$$k = \cosh\left(\frac{\pi R'(t)}{4R(t)}\right).$$

Here $R(t)$ is Legendre's complete elliptic integral of the first kind and $R'(t)$ is the complementary integral of $R(t)$ (see [5,6,10]).

For the function $p_{k,\gamma}(z)$ given by (1.3), let $\mathcal{P}\big(p_{k,\gamma}(z)\big)$ denote the class of functions $p(z)$ which are analytic in $\mathbb{U}$ such that

$$p(0) = 1 \qquad \text{and} \qquad p(z) \prec p_{k,\gamma}(z) \qquad (z \in \mathbb{U}).$$

We note that

$$p_k(z) := p_{k,1}(z) \qquad \text{and} \qquad \mathcal{P}\big(p_{k,1}(z)\big) =: \mathcal{P}\big(p_k(z)\big).$$

It can easily be seen that

$$\mathcal{P}\big(p_k(z)\big) \subset \mathcal{P}(\beta_1),$$

where

$$\mathcal{P}(\beta_1) := \{q : q \in \mathcal{A},\ q(0) = 1 \quad \text{and} \quad \Re[q(z)] > \beta_1 \quad (z \in \mathbb{U};\ 0 \leqq \beta_1 < 1)\}$$

with

$$\beta_1 = \frac{k}{1+k}. \tag{1.4}$$

For $p \in \mathcal{P}\big(p_k(z)\big)$, we have

$$|\arg[p(z)]| \leqq \frac{\sigma\pi}{2} \qquad (0 < \sigma \leqq 1;\ z \in \mathbb{U})$$