Applied Mathematics and Computation 285 (2016) 195–203

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Cosmological solutions in modified gravity with monomial nonlocality

Ivan Dimitrijevic*

Faculty of Mathematics, University of Belgrade, Studentski trg 16, Belgrade 11000, Serbia

ARTICLE INFO

MSC:
83D05
83F05
53B50

Keywords:
Modified gravity
Nonlocal gravity
Cosmological solutions

ABSTRACT

We consider cosmological properties of modified gravity with nonlocal term $R^p\mathcal{F}(\Box)R^q$ in its Lagrangian. Equations of motion are presented. For the flat FLRW metric, and some particular values of natural numbers p and q cosmological solutions of the form $a(t) = Ce^{-\frac{\gamma}{12}t^2}$ are found.

1. Introduction

Modern theory of gravity is general theory of relativity (GR), which was founded by Einstein one hundred years ago and has been successfully confirmed for the Solar System. It is given by the Einstein equations of motion for gravitational field $g_{\mu\nu}$: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$, which can be derived from the Einstein–Hilbert action $S = \frac{1}{16\pi G}\int R\sqrt{-g}d^4x + \int \mathcal{L}_{mat}\sqrt{-g}d^4x$, where $g = det(g_{\mu\nu})$ and units are chosen in such way that $c = 1$.

Despite all its successes, GR is not a final theory of gravity. There are many its modifications, which are motivated by quantum gravity, string theory, astrophysics and cosmology (for a review, see [1]). One of very promising directions of research is *nonlocal modified gravity* and its applications to cosmology (as a review, see [2,3]). To solve cosmological Big Bang singularity, nonlocal gravity with replacement $R \to R + CR\mathcal{F}(\Box)R$ in the Einstein–Hilbert action was proposed in [4]. This nonlocal model is further elaborated is the series of papers [5–10].

In this paper we consider the action

$$S_{pq} = \int \left(\frac{R - 2\Lambda}{16\pi G} + R^p\mathcal{F}\left(\frac{\Box}{M^2}\right)R^q \right)\sqrt{-g}d^4x \tag{1}$$

where R is scalar curvature, $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n\Box^n$ is an analytic function of the d'Alembert–Beltrami operator $\Box = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$, $g = det(g_{\mu\nu})$, M is a characteristic scale and p and q are natural numbers. For simplicity we take $M = 1$. At the end of this paper we briefly discuss the limit when $M \to +\infty$. In the paper [11] action (1) was introduced and constant scalar curvature cosmological solutions were obtained. Also, perturbations around de Sitter background were discussed in [12].

* Tel.: +381 641748310.
E-mail address: ivand@math.rs

http://dx.doi.org/10.1016/j.amc.2016.03.030

2. Equations of motion

Variation of the action (1) with respect to metric yields the equations of motion in the form

$$-\frac{1}{2}g_{\mu\nu}R^p\mathcal{F}(\Box)R^q + R_{\mu\nu}W_{pq} - K_{\mu\nu}W_{pq} + \frac{1}{2}\Omega_{pq\,\mu\nu} = -\frac{G_{\mu\nu}+\Lambda g_{\mu\nu}}{16\pi G}, \tag{2}$$

where

$$\begin{aligned} W_{pq} &= pR^{p-1}\mathcal{F}(\Box)R^q + qR^{q-1}\mathcal{F}(\Box)R^p,\\ K_{\mu\nu} &= \nabla_\mu\nabla_\nu - g_{\mu\nu}\Box,\\ \Omega_{pq\,\mu\nu} &= \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1}\Big(g_{\mu\nu}\nabla^\lambda\Box^l R^p\nabla_\lambda\Box^{n-1-l}R^q + g_{\mu\nu}\Box^l R^p\Box^{n-l}R^q - 2\nabla_\mu\Box^l R^p\nabla_\nu\Box^{n-1-l}R^q\Big), \end{aligned} \tag{3}$$

Detailed derivation of the above equations can be found in [11].

In this paper Friedmann–Lemaître–Robertson–Walker (FLRW) metric $ds^2 = -dt^2 + a^2(t)\big(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\big)$ is used. The signature of a metric is (1, 3) and the sign of a curvature tensor is chosen such that

$$R^\beta_{\mu\nu\alpha} = \partial_\nu\Gamma^\beta_{\mu\alpha} - \partial_\mu\Gamma^\beta_{\nu\alpha} + \Gamma^\lambda_{\mu\alpha}\Gamma^\beta_{\nu\lambda} - \Gamma^\lambda_{\nu\alpha}\Gamma^\beta_{\mu\lambda}, \tag{4}$$

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}. \tag{5}$$

Scalar curvature is $R = R_{\mu\nu}g^{\mu\nu} = 6(\frac{\ddot a}{a} + \frac{\dot a^2}{a^2})$ and $\Box h(t) = -\partial_t^2 h(t) - 3H\partial_t h(t)$, where $H = \frac{\dot a}{a}$ is the Hubble parameter.

Lemma 2.1. *If the metric is chosen to be FLRW, then the system (2) has two linearly independent equations.*

Proof. Note, that for the functions that only depend on time we have $K_{\mu\nu} = 0$ when $\mu \neq \nu$. Also $\Omega_{pq\,\mu\nu} = 0$ for $\mu \neq \nu$.It means that system (2) has four nontrivial equations. Equations with indices $\mu\nu$ equal to 11, 22 and 33 can be rewritten as

$$\begin{aligned} &16\pi G g_{ii}\Bigg(-\frac{1}{2}R^p\mathcal{F}(\Box)R^q + \left(\frac{\ddot a}{a} + 2\left(\frac{\dot a}{a}\right)^2\right)W_{pq} - \left(\ddot W_{pq} + 2\frac{\dot a}{a}\dot W_{pq}\right)\\ &+\frac{1}{2}\sum_{n=1}^{\infty} f_n\sum_{l=0}^{n-1}\left(\nabla^\lambda\Box^l R^p\nabla_\lambda\Box^{n-1-l}R^q + \Box^l R^p\Box^{n-l}R^q\right)\Bigg) = g_{ii}\left(\frac{2\ddot a a + \dot a^2}{a} - \Lambda\right). \end{aligned} \tag{6}$$

These equations are clearly proportional to each other and thus we have altogether two independent equations. The most convenient choice is to use trace and 00 equations, which are respectively

$$-2R^p\mathcal{F}(\Box)R^q + RW_{pq} + 3\Box W_{pq} + \frac{1}{2}\Omega_{pq} = \frac{R-4\Lambda}{16\pi G}, \tag{7}$$

$$\frac{1}{2}R^p\mathcal{F}(\Box)R^q + R_{00}W_{pq} - K_{00}W_{pq} + \frac{1}{2}\Omega_{pq\,00} = -\frac{G_{00}-\Lambda}{16\pi G}, \tag{8}$$

$$\Omega_{pq} = g^{\mu\nu}\Omega_{pq\,\mu\nu}. \tag{9}$$

□

At first, we investigate how does Eq. (2) change when parameters p and q replace their places in the action (1). To this end, the following lemma holds.

Lemma 2.2. *If we consider actions S_{pq}, given in (1), and S_{qp}. The corresponding equations of motion are equivalent.*

Proof. Equations of motion, given by Eq. (2), for actions S_{pq} and S_{qp} read

$$-\frac{1}{2}g_{\mu\nu}R^p\mathcal{F}(\Box)R^q + R_{\mu\nu}W_{pq} - K_{\mu\nu}W_{pq} + \frac{1}{2}\Omega_{pq\,\mu\nu} = -\frac{G_{\mu\nu}+\Lambda g_{\mu\nu}}{16\pi G}, \tag{10}$$

$$-\frac{1}{2}g_{\mu\nu}R^q\mathcal{F}(\Box)R^p + R_{\mu\nu}W_{qp} - K_{\mu\nu}W_{qp} + \frac{1}{2}\Omega_{qp\,\mu\nu} = -\frac{G_{\mu\nu}+\Lambda g_{\mu\nu}}{16\pi G}. \tag{11}$$

Since W_{pq} and W_{qp} coincide, subtraction of the last two equation yields

$$(R^p\mathcal{F}(\Box)R^q - R^q\mathcal{F}(\Box)R^p) = \sum_{n=1}^{\infty} f_n\sum_{l=0}^{n-1}\left(\Box^l R^p\Box^{n-l}R^q - \Box^l R^q\Box^{n-l}R^p\right). \tag{12}$$