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Numerical solution of Volterra–Fredholm integral equations via modification of hat functions

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ABSTRACT

In this paper, a new numerical approach is developed for solving linear and nonlinear Volterra–Fredholm integral equations. The fundamental structure of the presented method is based on the modification of hat functions. The properties of modification of hat functions (MHFs) are first presented. After implementation of our scheme, the solution of the main problem would be transformed into the solution of a system of linear or nonlinear algebraic equations. Also, an error analysis is provided under several mild conditions. In addition, examples are presented to illustrate the pertinent features of the method and the results are discussed.

1. Introduction

The Volterra–Fredholm integral equations arise from parabolic boundary value problems, from the mathematical modelling of the spatio-temporal development of an epidemic, and from various physical and biological models. The linear and nonlinear Volterra–Fredholm integral equations appear in the following form

$$f(x) = g(x) + \lambda_1 \int_0^x k_1(x, y) f(y) dy + \lambda_2 \int_0^1 k_2(x, y) f(y) dy, \quad x \in D = [0, 1], \tag{1}$$

and

$$f(x) = g(x) + \lambda_1 \int_0^x k_1(x, y) U_1(f(y)) dy + \lambda_2 \int_0^1 k_2(x, y) U_2(f(y)) dy, \quad x \in D, \tag{2}$$

respectively, where λ_1 and λ_2 are arbitrary integers, $g(x)$, $U_1(f(x))$, $U_2(f(x)) \in C^3(D)$ and $k_1(x, y)$, $k_2(x, y) \in C^3(D \times D)$ are known functions and $f(x) \in C^3(D)$ is an unknown function. We assume that (1) and (2) have a unique solution $f(x)$ [1–6].

Several numerical methods such as Taylor series and the Adomian decomposition methods [7,8], the modified decomposition methods [9] and other methods [10–12] have been used for solving linear Volterra–Fredholm integral equations. In recent years, many different method have been used to estimate the solution of nonlinear Volterra–Fredholm integral equations, such as collocation methods [6], Taylor polynomial methods [13], homotopy perturbation method [14], Triangular functions methods [15], rationalized Haar functions methods [16,17], wavelets methods [18] and other methods [19–21]. The essential features of the nonlinear case are of wide applicable [13]. Recently, Babolian et al. [22] have given a new method

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for solving systems of linear or nonlinear Fredholm integral equations of the second kind by hat basis functions. In this paper, we develop hat functions method and used this method for solving linear and nonlinear Volterra–Fredholm integral equations.

The paper is organized as follows: In Section 2, we describe MHFs and their properties. In Section 3, we will apply these sets of MHFs for approximating the solution of linear and nonlinear Volterra–Fredholm integral equations. In Section 4, convergence analysis of the present method is proved. Numerical results are given in Section 5 to illustrate the efficiency and accuracy of our algorithms. Finally , Section 6 concludes the paper.

2. MHFs and their properties

In this section, we give some basic definitions and properties of MHFs.

Definition. An $(m+1)$-set of MHFs consists of $(m+1)$ functions which are defined as follows [23]

$$h_0(x) = \begin{cases} \frac{1}{2h^2}(x-h)(x-2h) & 0 \leqslant x \leqslant 2h, \\ 0 & \text{otherwise}, \end{cases}$$

if i is odd and $1 \leqslant i \leqslant m-1$,

$$h_i(x) = \begin{cases} \frac{-1}{h^2}(x-(i-1)h)(x-(i+1)h) & (i-1)h \leqslant x \leqslant (i+1)h, \\ 0 & \text{otherwise}, \end{cases}$$

if i is even and $2 \leqslant i \leqslant m-2$,

$$h_i(x) = \begin{cases} \frac{1}{2h^2}(x-(i-1)h)(x-(i-2)h) & (i-2)h \leqslant x \leqslant ih, \\ \frac{1}{2h^2}(x-(i+1)h)(x-(i+2)h) & ih \leqslant x \leqslant (i+2)h, \\ 0 & \text{otherwise}, \end{cases}$$

and

$$h_m(x) = \begin{cases} \frac{1}{2h^2}(x-(1-h))(x-(1-2h)) & 1-2h \leqslant x \leqslant 1, \\ 0 & \text{otherwise}, \end{cases}$$

where $m \geqslant 2$ is an even integer and $h = \frac{1}{m}$.

According to definition of MHFs, we have

$$h_i(jh) = \begin{cases} 1 & i = j, \\ 0 & i \neq j, \end{cases} \tag{3}$$

$$h_i(x)h_j(x) = \begin{cases} 0 & i \text{ is even and } |i-j| \geqslant 3, \\ 0 & i \text{ is odd and } |i-j| \geqslant 2, \end{cases} \tag{4}$$

and

$$\sum_{i=0}^{m} h_i(x) = 1.$$

Let us write the MHFs vector $H(x)$ as follows

$$H(x) = [h_0(x), h_1(x), \ldots, h_m(x)]^T; \quad x \in D. \tag{5}$$

An arbitrary function $f(x)$ on D can be expanded by the MHFs as [23]

$$f(x) \simeq F^T H(x) = H^T(x) F,$$

where

$$F = [f_0, f_1, \ldots, f_m]^T,$$

and

$$f_i = f(ih), \quad i = 0, 1, \ldots, m.$$

Similarly an arbitrary function with two variables, $k(x, y)$ on $D \times D$ may be approximated by MHFs as follows:

$$k(x, y) \simeq H^T(x) K H(y),$$

where $H(x)$ and $H(y)$ are MHFs $(m+1)$-vector and K is the $(m+1) \times (m+1)$ MHFs coefficients matrix.

According to (3) and expanding $\int_0^x h_i(y)dy, \quad i = 0, 1, \ldots, m$ by MHFs, integration of vector $H(x)$ defined in (5) can be expressed as

$$\int_0^x H(y)dy \simeq P_1 H(x), \tag{6}$$