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Hybrid stabilization and synchronization of nonlinear systems with unbounded delays

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ABSTRACT

The stabilization of nonlinear systems with bounded and unbounded time-delays via hybrid control is studied. We investigate and identify switching rules for which stabilization can be verified *a priori*. When this approach is inadequate, stabilizing state-dependent switching rules are constructed. This method is based on partitioning the state-space into switching regions. Unwanted physical behavior, such as chattering and Zeno behavior, is avoided. Sufficient conditions are established using Razumikhin-like theorems. The theoretical results provide insight into how hybrid control strategies can be constructed to synchronize a class of nonlinear systems with unbounded delay. The findings are illustrated through numerical simulations.

1. Introduction

Delay-differential equations arise frequently in modeling various physical and biological phenomena, and, as a result, the theory and applications of systems with bounded and unbounded time-delays have emerged as an important area of research [1]. Applications can be found in infectious disease models with age-dependent population mixing [2], vector-borne disease models with finite incubation times [3,4], biological and artificial neural networks [5], biological population models, grazing systems, wave propagation, nuclear reactors, large-scale systems [1], heat flow problems, and chemical oscillations [6]. Synchronization problems with time-delays, in which two or more dynamical system states are synchronized (e.g., the error between a drive and response system is controlled to zero [7]), arise in consensus problems in multi-agent networks [8,9], chaotic systems [10,11], secure communications [12,13], and neural networks [14,15].

The main focus of the present article is the stabilization of nonlinear systems with time-delays using hybrid control. This problem is studied using a switched and impulsive system formulation, which combines continuous/discrete dynamics with logic-based switching. Switched systems exhibit unintuitive stability behavior such as the instability of a switched system comprised entirely of stable subsystems (e.g., see [16]). Finding classes of switching rules which guarantee stability is a major area of research. Concepts like dwell-time switching and multiple Lyapunov functions are used to guarantee stability in this case (e.g., see [7,16–19]). Stability of switched time-delay systems under classes of dwell-time switching rules has been studied some in the literature (e.g., see [20–23]). For background on switched systems, see [16,17,24–31] and the references therein.

A switched system composed entirely of unstable subsystems can also exhibit stability (e.g., see [16]), which naturally leads to the idea of using switching control to stabilize a system. There are a number of reasons why switching control is

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desirable, or even required, over continuous control [16,24,30]: continuous control may not be possible because of the nature of the problem, continuous control cannot be implemented because of sensor or actuator limitations, continuous control cannot be found because of uncertainty in the model, an appropriate switching control is easier to find, or the performance is improved under switching control. Authors have investigated the stabilization of unstable systems via high-frequency switching (e.g., see [32–36]). Alternatively, Wicks et al. [37] were the first to construct a state-dependent switching rule to stabilize a linear system. This approach was extended to linear systems with time-delays [38], nonlinear systems [39], and has been further investigated in, for example, [40–46].

The investigation here is motivated by a synchronization problem and the results found are applicable to more general stabilization problems. In particular, the main objective of the present manuscript is to analyze nonlinear switched systems with time-delays, including unbounded delay, and impulsive effects. This work extends the current literature by making contributions to the two areas of research outlined above: (i) finding classes of switching rules guaranteeing stability; and (ii) constructing switching rules to stabilize a system. In studying the first problem, dwell-time, average dwell-time, and periodic switching rules are found which ensure stabilization (and hence synchronization in the original context) in the presence of disturbance impulsive effects when each subsystem is stable. On the other hand, stability is also proved when stabilizing impulsive effects (which can be considered as impulsive control) are present and each subsystem is unstable. The switching rule is either given beforehand (if the problem is viewed as a naturally switching system) or is designed beforehand (if the problem is viewed from a control perspective) and is verified *a priori* to lead to stability using Razumikhin-like theorems.

For the second type of problem, the state-space is partitioned into different regions which in turn dictate the construction of a state-dependent switching rule. This can be viewed as a closed-loop hybrid control problem in which the user switches between different state feedback controllers available for use. Theoretical results are found guaranteeing stabilization for both stabilizing impulses and disturbance impulses. Hence, the state-dependent stabilization algorithms (e.g., minimum rule, wandering rule, and generalized rule [39]) are extended to nonlinear systems with time-delays while avoiding unwanted switching behavior (e.g., chattering and Zeno behavior). As a consequence of these investigations, the motivating synchronization problem is addressed for a class of nonlinear systems with unbounded delay.

This paper is organized as follows: in Section 2, preliminaries are given and the problem is formulated. The stabilization of nonlinear systems with unbounded time-delay via dwell-time satisfying switching control is studied in Section 3. In Section 4, the state-dependent switching stabilization algorithms are constructed and the main stability results are presented and proved. The theory is applied to a synchronization problem in Section 5. In Section 6, numerical simulations are used to illustrate the results. Conclusions and future directions are given in Section 7.

2. Problem formulation

Let $\mathbb{R}_+$ denote the set of nonnegative real numbers, let $\mathbb{Z}$ denote the set of integers, let $\mathbb{N}$ denote the set of positive integers, and let $\mathbb{R}^n$ denote the Euclidean space of n-dimensions (equipped with the Euclidean norm $\|\cdot\|$). In view of the potential applications in communication systems, time series analyses, and chaotic systems, Guan et al. [7] considered the following general synchronization synthesis problem: given a drive system

$$\dot{x}(t) = Ax(t) + f(t, x(t)),$$

with $A \in \mathbb{R}^{n\times n}$, $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$, and $x(t) \in \mathbb{R}^n$, and a response system

$$\dot{y}(t) = Ay(t) + f(t, y(t)) + u,$$

where $y(t) \in \mathbb{R}^n$, design a hybrid controller u which synchronizes the two systems (i.e., $\lim_{t\to\infty}\|y(t) - x(t)\| = 0$). The authors constructed a hybrid controller $u = u_1 + u_2$, where u_1 is a switching control and u_2 is an impulsive control:

$$u_1(t) := \textstyle\sum_{k=1}^{\infty} B_{1k}[y(t) - x(t)]\mathbf{1}_{\mathcal{I}_k}(t), \quad u_2(t) := \sum_{k=1}^{\infty} B_{2k}[y(t) - x(t)]\delta(t - t_k^-),$$

with $B_{1k}, B_{2k} \in \mathbb{R}^{n\times n}$, switching times $t_0 < t_1 < \ldots < t_{k-1} < t_k < \ldots$, and switching intervals $\mathcal{I}_k := [t_{k-1}, t_k)$. Here $\mathbf{1}_{\mathcal{I}_k}(\cdot)$ is the indicator function and $\delta(\cdot)$ is the generalized Dirac delta function.

Given the initial time $t_0 \in \mathbb{R}_+$ and initial conditions $x_0 := x(t_0) \in \mathbb{R}^n$, $y_0 := y(t_0) \in \mathbb{R}^n$, the synchronization error, $e := y - x$, is governed by the system

$$\begin{cases} \dot{e}(t) = [A + B_{1k}]e(t) + f(t, y(t)) - f(t, x(t)), & t \in \mathcal{I}_k, \\ e(t) = [I + B_{2k}]e(t^-), & t = t_k, \\ e(t_0) = y_0 - x_0, & k = 1, 2, \ldots \end{cases}$$

To develop conditions guaranteeing synchronization, Guan et al. analyzed the stability properties of the following hybrid switched and impulsive system:

$$\begin{cases} \dot{x}(t) = A_\sigma x(t) + F_\sigma(t, x(t)), & t \neq t_k \\ \Delta x(t) = B_\sigma x(t^-), & t = t_k, \\ x(t_0) = x_0, & k = 1, 2, \ldots, \end{cases}$$

where $\Delta x := x(t) - x(t^-)$ represents a sudden jump in the system state and $\sigma \in \mathcal{S}$ is called the switching rule, where the set $\mathcal{S}$ is defined as follows.