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Fitted reproducing kernel method for singularly perturbed delay initial value problems

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ABSTRACT

In this paper, a fitted reproducing kernel is proposed for solving singularly perturbed initial value problems. Based on it, an effective numerical method is proposed for a class of singularly perturbed delay initial value problem. Numerical results show that the present reproducing kernel is superior to the usual reproducing kernel for solving singularly perturbed initial value problems.

1. Introduction

Singularly perturbed delay initial value problems are at a very important position in science and engineering study field and have widely applied value. However, it is difficult to find exact solutions of most of singularly perturbed delay initial value problems. So the study of numerical solution methods play an important role in this field.

Recently, some authors gave some numerical methods for singularly perturbed delay initial value problems and boundary value problems in [1–3] and [4–10], respectively. The reproducing kernel method(RKM) was proposed originally by Cui, Geng et al. [11–13] for solving linear operator equations. In [14–21], the RKM was applied to solve some perturbed delay initial and boundary value problems.

The key to using the RKM is the construction of reproducing kernels. Recently, the reproducing kernel usually used in the RKM taking the polynomial form. However, based on the reproducing kernel with polynomial form, the RKM can not yield good numerical results for singularly perturbed initial value problems. To expand the scope of application areas of the RKM, we try to find fitted reproducing kernel for singularly perturbed initial value problems.

Consider the following singularly perturbed delay differential problem:

$$\begin{cases} \varepsilon u'(t) + a(t)u(t) + b(t)u(t-r) = f(t), t \in I, \\ u(t) = \varphi(t), t \in I_0 = (-r, 0], \end{cases} \tag{1.1}$$

where $0 < \epsilon \ll 1$, r is a constant delay, $I = (0, T] = \bigcup_{p=1}^{m} I_p,\ I_p = (t_{p-1}, t_p],\ t_p = p\, r,\ a(t) \ge \alpha > 0$, $b(t)$, $\varphi(t)$ and $f(t)$ are assumed to be sufficiently smooth functions such that the solution to (1.1) has boundary layers on the right side of each point $t = t_p (p = 1, 2, \ldots, m-1)$.

2. Construction of reproducing kernels

In this section, we will construct a fitted reproducing kernel.

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Definition 2.1. Let $W^2[a,b]=\{u(x)|u'(x)$ is an absolutely continuous real value function, $u''(x)\in L^2[a,b], u(a)=0\}$.The inner product and norm in $W^2[a,b]$ are defined as follows:

$$(u,v)_2=u(a)v(a)+u'(a)v'(a)+\int_a^b[u''(x)+\lambda u'(x)][v''(x)+\lambda v'(x)]dx \tag{2.1}$$

and

$$\| u \|_2=\sqrt{(u,u)_2},$$

where λ is a positive constant.

Theorem 2.1. *The space $W^2[a,b]$ is a reproducing kernel space and its reproducing kernel $k(x,y)$ is*

$$k(x,y)=\begin{cases} g_1(x,y), y\le x,\\ g_1(y,x), y>x,\end{cases} \tag{2.2}$$

where $g_1(x,t)=\frac{e^{\lambda(-(x+y))}(2((y+1-a)\lambda-1)e^{\lambda(x+y)}-2(\lambda-1)e^{(x+a)\lambda}-2(\lambda-1)e^{(y+a)\lambda}-e^{2y\lambda}+(2\lambda-1)e^{2a\lambda})}{2\lambda^3}$.

Proof. By [13], we can prove $W^2[a,b]$ is a reproducing kernel space. Here we only give the method of finding the reproducing kernel of $W^2[a,b]$.

By applying integrations by parts, we find that

$$\begin{aligned}(u(y),k(x,y))_2=&\int_a^b u(y)[k''''(x,y)-\lambda^2k''(x,y)]dy+u'(b)[k''(x,b)+\lambda k'(x,b)]-u'(a)[k''(x,a)\\ &+(\lambda-1)k'(x,a)]-u(b)[k'''(x,b)-\lambda^2k''(x,b)]+u(a)[k''(x,a)-\lambda^2k'(x,a)],\end{aligned} \tag{2.3}$$

where the prime denotes the derivative with respect to y.

From the fact that $k(x,y), u(y)\in W^2[a,b]$, clearly,

$$k(x,a)=0,\ \ u(a)=0. \tag{2.4}$$

Let

$$k''(x,b)+\lambda k'(x,b)=0,\ \ k''(x,a)+(\lambda-1)k'(x,a)=0,\ \ k'''(x,b)-\lambda^2k''(x,b)=0. \tag{2.5}$$

If

$$k''''(x,y)-\lambda^2k''(x,y)=\delta(y-x), \tag{2.6}$$

we must have

$$(u(y),k(x,y))_2=u(x).$$

From (2.6), we get

$$k(x,x-0)=k(x,x+0), k'(x,x-0)=k'(x,x+0), k''(x,x-0)=k''(x,x+0). \tag{2.7}$$

Furthermore, integrating (2.6) from $x-\varepsilon$ to $x+\varepsilon$ with respect to y and letting $\varepsilon\to 0$, it follows that

$$k'''(x,x+0)-k'''(x,x-0)=1. \tag{2.8}$$

Characteristic equation of (2.6) is given by

$$r^4-\lambda^2r^2=0.$$

Obviously, the characteristic values are $r=0, r=\lambda, r=-\lambda$. Therefore, we let

$$k(x,y)=\begin{cases} c_1+c_2y+c_3e^{\lambda y}+c_4e^{-\lambda y}, & y\le x,\\ d_1+c_2y+d_3e^{\lambda y}+d_4e^{-\lambda y}, & y>x.\end{cases} \tag{2.9}$$

From (2.4),(2.5), (2.7) and (2.8), the unknown constants of $k(x,y)$ in (2.9) can be determined easily. □

3. Brief introduction of the fitted reproducing kernel method

In this section, we introduce the fitted reproducing kernel method for the following singularly perturbed initial value problems:

$$\begin{cases}\varepsilon u'(t)+a(t)u(t)=p(t), t\in(a,b],\\ u(a)=\alpha.\end{cases} \tag{3.1}$$

Introducing $v(t)=u(t)-\alpha$, (3.1) become an equation with homogeneous initial condition

$$\begin{cases}\varepsilon v'(t)+a(t)v(t)=p(t)-a(t)\alpha=q(t), t\in(a,b],\\ v(a)=0.\end{cases} \tag{3.2}$$