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Egalitarian solutions to multiperson social dilemmas in populations

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ABSTRACT

We consider a class of multiperson social dilemma games played in large populations. In particular, the popular games, such as for example the N-person Prisoner's Dilemma, the Public Goods, the Tragedy of the Commons, the Volunteer's Dilemma, and the Assurance game, are included in the proposed frame. The evolution of such populations is assumed to be governed by the replicator equations. We show that the egalitarian distribution of the social welfare generated in the multiperson social dilemma games fosters the long run cooperation in such populations.

1. Introduction

The problem of the evolution of cooperation is one of the most important problems in social sciences. A large body of the relevant research is based on social dilemmas. Social dilemmas describe situations in which selfish decisions by autonomous individuals jointly result in group inefficient outcomes. The notion dilemma comes from the fact that acting in one's self-interest is tempting to every individual involved, even though all individuals would benefits from acting in the group interest. Social dilemmas have attracted a great deal of interest of scholars in various branches of science, among others in psychology, sociology, economy, politics, biology, cf. e.g., [1–9] and references cited therein.

The formal approach to social dilemmas is provided by game theory. The corresponding models, i.e. social dilemma games, were studied by many scholars, cf. e.g., [2,10–13].

In the context of N-person strategic games with two strategies: cooperate, and defect, various axiomatization schemes were proposed, cf. e.g. [2,12,14]. In particular, the definition of Liebrand [12] extended the set of social dilemmas from the PD game, studied as a paradigm for the evolution of cooperation, to other two-player games: the Chicken (Snowdrift) game, and the Stag Hunt (Assurance) game. In [14] a general classification of multiperson social dilemma games with two strategies has been proposed.

There are various approaches to overcome "the dilemma of social dilemmas" in the frame of strategic game theory. For example, one can add to a formal model additional features which enrich the model and lead to the change of the solution of the game, cf. e.g., [15–19] and references cited therein. Another approach is to study social dilemma games in a broader context, such as games on graphs, cf. e.g., [20,21], or social dilemma games in populations, cf. e.g., [22–26] and references cited therein. In particular, multiperson social dilemmas in structured populations have been studied frequently in the recent past, cf. e.g., [27–29].

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Evolutionary dynamics has been applied to solve social dilemmas in populations in various contexts, cf. for example [30–33] and references therein. In this paper we consider a class of multiplayer social dilemma games played in large populations in the case when the players can choose one of two available strategies, cooperate or defect. The popular games, such as for example the N-person Prisoner's Dilemma (NPD), the Public Goods (PG), the Tragedy of the Commons (TC), the Volunteer's Dilemma, and the Assurance game, are solved using the proposed scheme. The evolution of such populations will be described by the replicator equations. The players are randomly matched in groups of the same cardinality at each instant of time, and play a multiperson social dilemma game. The individual payoffs of the players are collected together and redistributed between them, using an imposed or accepted procedure. In this note we use egalitarian procedure, in which each player receives the same share of the total welfare. The redistributed payoffs in the fully mixed population determine the choice of the strategies of the players in the next time step, based, for example, on imitation of the most successful strategies, cf. e.g. [22–24], where the application of the replicator equations to social systems has been discussed. We show that the egalitarian distribution of the social welfare generated in the multiperson social dilemma games fosters the long run cooperation in the considered populations.

In the next two sections we define the social dilemma games and the relevant framework of the evolutionary game theory, with dynamics described by the replicator equations, and prove the main results of fostering cooperation in the considered class of multiperson social dilemma games. In Section 4 we briefly discuss several open problems. Some extensions and an axiomatization of the considered social dilemma games are discussed in the appendices.

2. Multiplayer social dilemma games

We consider general N-person strategic games:

$$< \aleph, \{C, D\}, P_C, P_D >, \tag{1}$$

where $\aleph$ is the set of $N \geq 2$ identical individuals, who can choose one of two strategies: C-cooperate, or D-defect, and receive strategy dependent payoffs: $P_C(n),\ n \in \{1, \dots, N\}$ for an individual who plays strategy C, and $P_D(n),\ n \in \{0, 1, \dots, N-1\}$ for that who plays strategy D, where n is the number of players who choose strategy C. The associated payoff matrix reads

	$C...C$	$C...CD$	...	$CD...D$	$D...D$
C	$P_C(N)$	$P_C(N-1)$	...	$P_C(2)$	$P_C(1)$
D	$P_D(N-1)$	$P_D(N-2)$	...	$P_D(1)$	$P_D(0)$

(2)

where e.g., $C\dots CD$ stands for the set of $N-2$ C–players and one D-player. The game defined by the payoff matrix (2) will be called the **initial game**. Occasionally, for brevity we shall use vector notation, for example for $N = 3$ the matrix (2) will be replaced by the vector $[P_C(3), P_C(2), P_C(1), P_D(2), P_D(1), P_D(0)]$.

The social welfare of the coalition of N-players with n C-players is defined as the sum of the payoffs of all the players:

$$T(n) = nP_C(n) + (N-n)P_D(n), \quad n = 1, ..., N-1, \ T(0) = NP_D(0), \quad T(N) = NP_C(N). \tag{3}$$

We consider a large population of individuals, entities, that at each instant of time are randomly matched into finite groups of the same cardinality $N \geq 2$. The agents are wired to one of two types of strategies: C or D. The strategy played is the unique feature that identifies the agent: the agents using the same strategy are indistinguishable.

In each group the agents interact. The interactions are described by an N-player strategic game (1). The payoffs of all N players are collected together and redistributed between them by an external party, power, or as the result of an a priori agreement, using an imposed or accepted procedure. The value $T(n)$ of the coalition of N players is redistributed between its members. Various procedures, based on the strength of the players defined by the coalitional game theory can be used, cf. [35]. We use the egalitarian procedure, in which each player receives the same share $\frac{1}{N}T(n)$ of the total welfare. The payoff matrix after redistribution reads

	$C...C$	$C...CD$	...	$CD...D$	$D...D$
C	$\hat{P}_C(N)$	$\hat{P}_C(N-1)$	...	$\hat{P}_C(2)$	$\hat{P}_C(1)$
D	$\hat{P}_D(N-1)$	$\hat{P}_D(N-2)$	...	$\hat{P}_D(1)$	$\hat{P}_D(0)$

(4)

where

$$\hat{P}_C(n) = \hat{P}_D(n) = \frac{1}{N}T(n), \quad n = 1, \dots, N-1, \tag{5}$$

$$\hat{P}_C(N) = P_C(N), \quad \hat{P}_D(0) = P_D(0). \tag{6}$$

The game defined by the payoff matrix (4) will be called the **redistributed game**.