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Modeling and analysis in a prey–predator system with commercial harvesting and double time delays

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ABSTRACT

A differential-algebraic prey–predator system with commercial harvesting on predator is proposed, where maturation delay for prey and gestation delay for predator are considered. Since commercial harvesting is dynamically influenced by variation of economic interest, we will investigate combined dynamic effects of double time delays and economic interest on population dynamics. Positivity of solutions and uniform persistence of system are studied. In the absence of time delay, by taking economic interest as bifurcation parameter, existence of singularity induced bifurcation is investigated based on differential-algebraic system theory. State feedback controllers are designed to eliminate singularity induced bifurcation and stabilize the proposed system around corresponding interior equilibrium. In the presence of double time delays, by analyzing associated characteristic transcendental equation, it is found that interior equilibrium loses local stability when double time delays cross corresponding critical values. According to Hopf bifurcation theorem for functional differential equation, existence of Hopf bifurcation is investigated as local stability switches. Based on normal form theory and center manifold theorem, directions of Hopf bifurcation and stability of the bifurcating periodic solutions are studied. Numerical simulations are carried out to show consistency with theoretical analysis.

1. Introduction

In recent decades, many research efforts have been put into investigation of population dynamics of prey–predator ecosystem [4,13,23,25,27]. When investigating such biological phenomena arising from prey–predator ecosystem, there are many factors which affect dynamical properties of biological and mathematical models, and one of the familiar nonlinear factors is functional response. A functional response in ecology is the intake rate of a consumer as a function of food density. It is associated with numerical response, which is reproduction rate of a consumer as a function of food density. Among widely used mathematical models in theoretical ecology, mathematical model with Holling–Tanner functional response plays a special role in view of interesting dynamics it possesses, and dynamics have been of interest to both

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applied mathematicians and ecologists [4,13,23,25,27]. The Holling–Tanner system is studied for its efficacy for describing prey–predator ecosystems in the real world like Canadian lynx/snowshoe hare [5], mite/spider mite [7,11], vole/weasel [14] ecosystem, and the Holling–Tanner system is generally governed by following differential system (1),

$$\begin{cases} \dot{X}(T) = s_1 X(T)\left(1 - \dfrac{X(T)}{K}\right) - \dfrac{l_1 X(T) Y(T)}{l_2 + X(T)}, \\ \dot{Y}(T) = s_2 Y(T)\left(1 - \dfrac{l_3 Y(T)}{X(T)}\right), \end{cases} \tag{1}$$

where the first equation shows that prey population $X(T)$ grows logistically with carrying capacity K and intrinsic growth rate s_1 in absence of predation, mathematical term $\frac{l_1 X(T)}{l_2 + X(T)}$ represents predation rate, number killed per predator population per time, which is also known as a Holling type II predator response [2,4,13,23,25,27]. l_1 is the maximum number of prey population that can be eaten per predator population per time and l_2 is half saturation constant at which predation rate achieves $\frac{l_2}{2}$. The second equation shows that predator population $Y(t)$ grows logistically with intrinsic growth rate s_2 and carrying capacity is proportional to prey population size. l_3 denotes the number of prey population required to support one predator population at equilibrium state.

Dynamical analysis of system (1) have been studied in [4,5,19,35,36]. By using Kolmogorov's theorem, May [4] investigates local stability of interior equilibrium and existence of stable limit cycles. Along with the line of this research, method proposed in [4] is simplified by Tanner [5]. Hsu and Huang [19] discuss some general conditions under which interior equilibrium is a stable node or focus and under which the system (1) possesses a stable limit cycle. Shi et al. [35] investigate a diffusive prey–predator system with modified Holling–Tanner functional response under homogeneous Neumann boundary condition, local and global asymptotic stability of unique positive constant equilibrium are obtained. In [36], an open question left in [19] is investigated, existence and uniqueness of stable limit cycles and Hopf bifurcations of Holling–Tanner prey–predator system are studied. Furthermore, some well known results observed and suggested from the real ecological systems in [5] are confirmed in [36].

Recently, traditional prey–predator model, which the functional response depends on prey population density only, have been challenged by several ecologists. There is a growing explicit biological and physiological evidence [15,17] that in many situations especially when predator population has to search for food (and therefore has to share or compete for food), a more suitable general prey–predator theory should be based on fact per capital predator population growth rate should be a function of ratio of prey to predator abundance. Based on the above analysis, a ratio-dependent prey–predator model with Holling–Tanner functional response is established in [30], and predation rate $\frac{l_1 X(T) Y(T)}{l_2 + X(T)}$ in the first equation of system (1) is replaced with Leslie–Gower term [3] $\frac{l_1 X(T) Y(T)}{l_2 Y(T) + X(T)}$, which is as follows:

$$\begin{cases} \dot{X}(T) = s_1 X(T)\left(1 - \dfrac{X(T)}{K}\right) - \dfrac{l_1 X(T) Y(T)}{l_2 Y(T) + X(T)}, \\ \dot{Y}(T) = s_2 Y(T)\left(1 - \dfrac{l_3 Y(T)}{X(T)}\right). \end{cases} \tag{2}$$

By using following transformations: $t = s_1 T, x = \frac{X}{K}, y = \frac{l_1 Y}{s_1 K}$, $\alpha = \frac{s_1 l_2}{l_1}$, $r = \frac{l_1}{l_3 s_1}$, $\beta = \frac{s_2 l_3}{l_1}$, system (2) is non-dimensionalized as follows:

$$\begin{cases} \dot{x}(t) = x(t)(1 - x(t)) - \dfrac{x(t) y(t)}{x(t) + \alpha y(t)}, \\ \dot{y}(t) = \beta y(t)\left(r - \dfrac{y(t)}{x(t)}\right). \end{cases} \tag{3}$$

where α, β and r are all positive constants. Liang and Pan [30] investigate global stability property and uniqueness of limit cycle of system (3).

Generally, in order to reflect interaction and coexistence mechanism of population depending on the past history, time delay is usually incorporated into mathematical models, which can be utilized to describe the hunting delay, maturation delay and gestation delay for population within prey–predator ecosystem [6,16,18]. Delay differential equation models are capable of generating rich, more effective and accurate dynamics compared to ordinary differential equation models when it is necessary to capture oscillatory dynamics [6,16,18]. Recently, many theoreticians and experimentalists have discussed dynamical behavior of prey–predator system with Holling–Tanner functional response, it reveals that time delay may cause the loss of stability and other complicated dynamical behavior such as the periodic structure and bifurcation phenomenon