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Fully developed flow of a drilling fluid between two rotating cylinders

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ABSTRACT

In this paper, we study the fully developed Couette flow of a drilling fluid, and explore the effects of concentration and shear-rate-dependent viscosity. The one-dimensional form of the governing equations, as well as the boundary conditions are made dimensionless and a parametric study is performed by varying the dimensionless numbers.

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1. Introduction

Even though drilling is an old technology, the basic fundamentals of drilling science and technology are not yet well-understood. To meet the current energy demands the drilling may need to be done at a greater depth (usually referred to as extreme drilling) and also in areas where climate condition is not so amenable. Among the many types of drilling, one can mention offshore and deep-ocean drilling, extended reach wells, horizontal wells, multi-branch wells, etc. [1]. Many researchers have been engaged in finding ways to improve the efficiency of these drilling operations. The classical drilling method is the rotary drilling technique [2], where a drill bit attached to the drill string rotates at a constant speed cutting the rock-type materials. These cuttings are then lifted off to the surface by a fluid which is circulated downward through the drill pipe and upward through the annular space between the rock and the pipe [3]. The fluid used in the drilling operation is often called the drilling fluid or the drilling mud. Much of the success of any drilling operation depends, to a large extent, on the performance of the drilling fluid. Hole stability, rate of penetration, loss of circulation, etc., depend on the rheological properties of the drilling fluid. Many studies have focused on determining the rheological parameters that are most useful for improving the hole efficiency [4].

In case of horizontal drilling, as mentioned by Caenn and Chillingar [5], hole cleaning and maintaining the integrity of the well bore are the main issues. Another important parameter, mentioned by Siginer and Bakhtiyavou [6] is the effect of the eccentricity of pipe on the flow of well bore fluids. For deep drillings, in addition to external factors such as temperature, pressure etc., contributing to instability problems in mudrocks, intrinsic factors such as permeability, deformation properties, etc., are also important factors related to mudrock instability [7]. Slawomriski [8], for example, indicated that even though the Bingham plastic model is often used in modeling the drilling fluids, some studies have shown that many of the drilling fluids are non-linear fluids with memory. Briscoe et al. [9] indicate that in high pressure and high temperature environment,

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Nomenclature

Symbol	Explanation
$\boldsymbol{b}$	body force vector
ϕ	concentration
$\boldsymbol{D}$	symmetric part of the velocity gradient
g	acceleration due to gravity
h	characteristic length
$\boldsymbol{I}$	identity tensor
$\boldsymbol{L}$	gradient of the velocity vector
n	power-law index
t	time
$\boldsymbol{T}$	Cauchy stress tensor
$\boldsymbol{v}_r$	reference velocity
ρ	bulk density
μ_r	reference viscosity
η	effective viscosity
θ	temperature
div	divergence operator
∇	gradient symbol

the yield stress and the plastic viscosity of the muds are also influenced by pressure and temperature. Al-Zuhrani [10] suggested a generalized shear-thinning model where the effect of yield stress are included.

The objective of the present paper is to study the fully developed Couette flow of a drilling fluid, and explore the effects of concentration and shear-rate-dependent viscosity. In the next section, the governing equations of motion are provided. Section 3 focuses on the constitutive relations for the stress tensor and the diffusive particle flux vector. In Section 4, we describe the geometry of the problem and provide the derivation for the one-dimensional form of the governing equations, as well as the boundary conditions. In Section 5, we outline the numerical scheme we have used. In Section 6, the numerical results are presented through a parametric study by varying the dimensionless numbers.

2. Governing equations

If the drilling fluid is treated as a single component (phase) material then, in the absence of any electro-magnetic effects, the governing equations of motion are the conservations of mass, linear momentum, angular momentum, concentration and the energy equation. If the drilling fluid is modeled as a multi-component material, then the governing equations should be given for all the components, and a multi-phase approach should be taken, this requires not only constitutive relations for each component, but also for the interactions among the components. [11–14]. In this paper, we assume that the drilling fluid can be treated as a single component non-homogenous fluid. As a result, the governing equations are [3,15]:

2.1. Conservation of mass

$$\frac{\partial \rho}{\partial t} + div(\rho \boldsymbol{v}) = 0 \tag{1}$$

where ρ is the density of the fluid, $\partial/\partial t$ is the partial derivative with respect to time, and $\boldsymbol{v}$ is the velocity vector. For an isochoric motion we have $div\ \boldsymbol{v} = 0$.

2.2. Conservation of linear momentum

$$\rho \frac{d\boldsymbol{v}}{dt} = div\boldsymbol{T} + \rho \boldsymbol{b} \tag{2}$$

where $\boldsymbol{b}$ is the body force vector, $\boldsymbol{T}$ is the Cauchy stress tensor, and d/dt is the total time derivative, given by $d(.)/dt = \partial(.)/\partial t + [grad(.)]\boldsymbol{v}$. The conservation of angular momentum indicates that in the absence of couple stresses the stress tensor is symmetric, that is, $\boldsymbol{T} = \boldsymbol{T}^{\boldsymbol{T}}$.

In suspension flows, the particle concentration is, in general, not constant and in many applications, it is necessary to have an additional equation, often called the *convection–diffusion* equation. Here, we use the particle concentration equation for ϕ as discussed in [15], based on [16], where