Applied Mathematics and Computation 281 (2016) 278–299

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

On four-point penalized Lagrange subdivision schemes

X. Si [a], J. Baccou [b,c], J. Liandrat [a,*]

[a] Centrale Marseille, I2M, UMR 7373, CNRS, Aix-Marseille Université, Marseille 13453, France
[b] Institut de Radioprotection et de Sûreté Nucléaire(IRSN), PSN-RES,SEMIA, LIMAR, Cadarache, Saint Paul-Lez-Durance 13115, France
[c] Laboratoire de Micromécanique et d'Intégrité des Structures (MIST), IRSN-CNRS-UMII, B.P. 3, Cadarache, Saint Paul-Lez-Durance 13115, France

ARTICLE INFO

Keywords:
Lagrange subdivision scheme
Kriging
Penalization
Data reconstruction
Gibbs phenomenon

ABSTRACT

This paper is devoted to the definition and analysis of new subdivision schemes called penalized Lagrange. Their construction is based on an original reformulation for the construction of the coefficients of the mask associated to the classical 4-points Lagrange interpolatory subdivision scheme: these coefficients can be formally interpreted as the solution of a linear system similar to the one resulting from the constrained minimization problem in Kriging theory which is commonly used for reconstruction in geostatistical studies. In such a framework, the introduction in the formulation of a so-called error variance can be viewed as a penalization of the oscillations of the coefficients. Following this idea, we propose to penalize the 4-points Lagrange system. This penalization transforms the interpolatory schemes into approximating ones with specific properties suitable for the subdivision of locally noisy or strongly oscillating data. According to a so-called penalization vector, a family of schemes can be generated. A full theoretical study is first performed to analyze this new type of non stationary subdivision schemes. Then, in the framework of position dependant penalization vector, several numerical tests are provided to point out the efficiency of these schemes compared to standard approaches.

1. Introduction

Since Deslaurier and Dubuc [6], Lagrange interpolatory subdivision schemes have been intensively used. Originally, they are designed for generating curves starting from initial control polygons by repeated refinements. These schemes and their extensions are working horses for multi-scale approximation and are used in many applications of numerical analysis including numerical solutions of partial differential equations, data reconstruction or image and signal analysis. Their convergence is well understood (at least when they remain linear). Smooth curves can then be generated, leading to efficient tools for approximation problems. Their major drawback lays in the existence of Gibbs oscillations for the limit curve when initial data exhibit strong gradients (see Fig. B.4 top left). These oscillations are connected to the oscillations of the coefficients involved in the Lagrange scheme (the set of these coefficients is called the mask of the scheme).

Adaption of the schemes to strong gradients is the motivation of a large family of recently developed schemes: these schemes can be position dependent, as in [1] where the monitoring of the position dependency is performed through a segmentation of the data performed a priori, or data dependent (and therefore non-linear) as in [4] where the adaption is performed at each application of the scheme, according to the data. In all these examples, the schemes remain interpolatory.

* Corresponding author. Tel: +33 4 91 05 47 49; Fax: +33 4 91 05 47 49.
E-mail address: jliandrat@centrale-marseille.fr, jacques.liandrat@centrale-marseille.fr (J. Liandrat).

http://dx.doi.org/10.1016/j.amc.2016.01.030

Our goal in this paper, is to introduce a new approach that allows to transform locally an interpolatory scheme into a non interpolatory one, more robust to jump discontinuities. According to the way this approach is applied, it can lead to position dependent schemes (as described in Section 5 of this paper) or to data dependent schemes, as it will be described in a forthcoming publication.

This paper is then devoted to a new approach of Lagrange schemes that allows to transform them locally into non interpolatory ones. The starting point is a recent paper, [2], where the mask of the Lagrange interpolatory subdivision schemes appeared as the limit mask of a non-stationary subdivision scheme based on Kriging approach. Kriging, [5], is a stochastic data modeling and reconstruction tool widely used in the framework of spatial data analysis. Its main advantages stand in the possibility to integrate in the reconstruction the spatial dependency of the available data and to quantify the precision of the reconstruction thanks to the model describing this dependency. When they are interpolatory, Kriging schemes are subject to Gibbs oscillations [10], in the vicinity of strong gradients. However, this drawback can be corrected introducing a so called error variance vector [3]. This vector can be interpreted as a penalization of the oscillations of the subdivision mask coefficients that appear, in the Kriging approach, as solution of an optimization procedure.

The new approach developed in this paper mimics the use of error variance in Kriging to generate new schemes derived from the Lagrange interpolatory framework. They will be called penalized Lagrange subdivision schemes. The paper is organized as follows: after a quick overview of binary subdivision schemes in Section 2, we establish in Section 3 the connection between the Lagrange and Kriging frameworks. It leads to the construction of penalized Lagrange subdivision schemes. This new type of schemes is fully analyzed in Section 4 in the case of a 4-point centered prediction. Then, in Section 5, it is coupled with a zone-dependent strategy in order to accurately reconstruct discontinuous or locally noisy data. The convergence of this new zone-dependent scheme is studied. Finally, Section 6 provides several numerical tests in order to point out the efficiency of these schemes compared to standard approaches.

2. Basic notations and results for binary subdivision schemes

2.1. Lagrange interpolatory subdivision scheme

A (univariate and binary) subdivision scheme S is defined by a linear operator $S : l^\infty(\mathbb{Z}) \to l^\infty(\mathbb{Z})$ constructed from a real-valued sequence $(a_k)_{k\in\mathbb{Z}}$ with finite support (called mask of S) such that

$$(f_k)_{k\in\mathbb{Z}} \in l^\infty(\mathbb{Z}) \mapsto ((Sf)_k)_{k\in\mathbb{Z}} \in l^\infty(\mathbb{Z}) \quad \text{with} \quad (Sf)_k = \sum_{l\in\mathbb{Z}} a_{k-2l} f_l.$$

The mask plays a key role in the subdivision process and there exist many works dealing with its construction. Among them, one can mention [4,8] and [1], that are devoted to interpolatory Lagrange-based schemes. These schemes use masks having $(l+r+1)$ non zero coefficients (a_0 and $a_{2k+1}, -l \le k \le r-1$) and interpolatory polynomials of degree $l+r-1$. The sets of non zero coefficients of even index (resp. odd index) of the mask are called the stencils of the scheme. For interpolatory schemes, the even indexed coefficients of the mask are characterized by $a_0 = 1,\ a_{2k} = 0$ for $k \neq 0$. In the case of Lagrange schemes the odd indexed coefficients of the mask are given by,

$$a_{2k+1} = L_k^{l,r}(-\frac{1}{2}), -l \le k \le r-1,$$

where $L_k^{l,r}$ is the elementary Lagrange interpolatory polynomial defined by:

$$L_k^{l,r}(x) = \Pi_{n=-l,n\neq k}^{r-1} \frac{x-n}{k-n}.$$

In this article, we will mainly consider the 4-point centered Lagrange interpolatory scheme corresponding to $l = r = 2$. It can be written as:

$$\begin{cases} (Sf)_{2k} = f_k, \\ (Sf)_{2k+1} = -\dfrac{1}{16} f_{k-1} + \dfrac{9}{16} f_k + \dfrac{9}{16} f_{k+1} - \dfrac{1}{16} f_{k+2}, \end{cases} \tag{1}$$

Subdivision is iterated from an initial sequence $(f_k^0)_{k\in\mathbb{Z}}$ to generate $(f_k^j)_{k\in\mathbb{Z}}$ for $j \ge 1$ as

$$f^{j+1} = Sf^j, j \ge 0.$$

Parameter j is called the scale parameter and is linked to the dyadic grid $X_j = \left\{k2^{-j}, k \in \mathbb{Z}\right\}$. If the coefficients of the masks are scale invariant, the scheme is stationary whereas it is non-stationary when they depend on j.

2.2. Convergence of subdivision schemes

The uniform convergence of subdivision schemes is made precise in the following definition.

Definition 1. The subdivision scheme S is said to be uniformly convergent if for any real sequence $\left(f_k^0\right)_{k\in\mathbb{Z}}$, there exists a continuous function f (called the limit function associated to f^0) such that: $\forall\epsilon, \exists J$ such that $\forall j \ge J,\ \| S^j f^0 - f\left(\frac{\cdot}{2^j}\right) \|_\infty \le \epsilon$.