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A two-grid block-centered finite difference method for nonlinear non-Fickian flow model

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ABSTRACT

In this paper, a two-grid block-centered finite difference scheme is introduced and analyzed to solve the nonlinear parabolic integro-differential equation arising in modeling non-Fickian flow in porous media. This method is considered where the nonlinear problem is solved only on a coarse grid of size H and a linear problem is solved on a fine grid of size h. Error estimates are established on non-uniform rectangular grid which show that the discrete $L^\infty(L^2)$ and $L^2(H^1)$ errors are $O(\triangle t + h^2 + H^3)$. Finally, some numerical experiments are presented to show the efficiency of the two-grid method and verify that the convergence rates are in agreement with the theoretical analysis.

1. Introduction

In this paper, we consider the nonlinear integro-differential equation arising in modeling non-Fickian flow in porous media.

Find $p = p(\boldsymbol{x}, t)$ such that

$$\begin{cases} \dfrac{\partial p}{\partial t} - \nabla \cdot \left(a(\boldsymbol{x}, t, p)\nabla p + \int_0^t b(\boldsymbol{x}, s, p)\nabla p(\boldsymbol{x}, s)ds \right) = f(\boldsymbol{x}, t, p), \boldsymbol{x} \in \Omega, t \in J, \\ \nabla p(\boldsymbol{x}, t) \cdot \boldsymbol{n} = 0, \quad \boldsymbol{x} \in \partial\Omega, t \in J, \\ p \mid_{t=0} = p_0(\boldsymbol{x}), \quad \boldsymbol{x} \in \Omega \cup \partial\Omega. \end{cases} \tag{1}$$

where Ω is a rectangular domain in R^2, $\boldsymbol{n}$ is the unit outward normal vector of the domain Ω. $J = (0, T]$, and T denotes the final time.

Problem (1) is very important in the transfer of reaction and passive contaminates in aquifers. This model is complicated by the history effect, which characterizes various mixing length growth of flow [1,2] and arises from many physical processes in which it is necessary to take into account the effects of memory due to the deficiency of the usual diffusion equations. As shown in [3,4], non-Fickian flow of fluid in porous media can be served as engineering model for nonlocal reactive transport. Besides, it can also demonstrate heat conduction with memory [3].

There are many papers on the numerical methods for this kind of problems. Ewing et al. [5] and Lin et al. [6] presented the finite volume methods for this problem. And finite element methods for this problem have been considered in [7]. Some

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numerical methods for integro-differential equations of parabolic and hyperbolic types have been demonstrated in [8]. And it is presented split least-squares finite element methods for non-Fickian flow in porous media in [9]. Moreover, mixed element methods for this problem are given in [10]. Wing and his coworkers presented the backward Euler mixed FEM and regularity of parabolic integro-differential equations [11]. Furthermore, it's considered a two-grid expanded mixed element method for nonlinear non-Fickian flow model [12].

Block-centered finite differences, sometimes called cell-centered finite differences, can be thought as the lowest order Raviart–Thomas mixed element method [13], with proper quadrature formulation. In [14], Wheeler presented the mixed finite elements for elliptic problems with tensor coefficients as cell-centered finite differences. And in 2012, a block-centered finite difference method for the Darcy–Forchheimer model was considered [15]. In [16,17] block-centered finite difference methods were developed.

However, as far as we know, there is no two-grid block-centered finite difference methods for nonlinear non-Fickian flow model. And we propose the corresponding algorithm in this paper. The idea for two-grid scheme is taken from Xu [18] and Wheeler with his coworkers [19]. Error estimates are established rigorously and carefully on non-uniform rectangular grid which show that the discrete $L^\infty(L^2)$ and $L^2(H^1)$ errors are $O(\triangle t + h^2 + H^3)$. Finally, some numerical experiments are presented to show the efficiency of the two-grid method and verify that the convergence rates are in agreement with the theoretical analysis. Moreover, we also give the numerical examples of the nonlinear implicit scheme to illustrate the efficiency of the two-grid block-centered finite difference method.

The paper is organized as follows. In Section 2 we give some preliminaries. In Section 3 we present a two-grid characteristic block-centered finite difference algorithm and corresponding error estimates. In Section 4 some numerical experiments are carried out, which show that the convergence rates are in agreement with the theoretical analysis. Finally, conclusions and extensions are drawn in the last section.

Throughout the paper we use C, with or without subscript, to denote a positive constant, which could have different values at different appearances.

2. Preliminaries

Firstly, in this section, we define some notations. Let $N > 0$ be a positive integer. Set $\triangle t = T/N, t_n = n\triangle t, for\ n \le N$.

Let $L^p(\Omega)$ be the standard Banach space with norm

$$\|v\|_{L^p(\Omega)} = \left(\int_\Omega |v|^p d\Omega\right)^{1/p}.$$

For simplicity, let $(\cdot,\cdot)$ denote the $L^2(\Omega)$ inner product. And $W_p^k(\Omega)$ be the standard Sobolev space

$$W_p^k(\Omega) = \{g : \|g\|_{W_p^k(\Omega)} < \infty\},$$

where

$$\|g\|_{W_p^k(\Omega)} = \left(\sum_{|\alpha|\le k} \|D^\alpha g\|^p_{L^p(\Omega)}\right)^{1/p}.$$

Let $S = L^2(\Omega)$ and $W = H(\Omega, div) = \{\boldsymbol{w} \in (L^2(\Omega))^d, \nabla\cdot\boldsymbol{w} \in L^2(\Omega)\}$. And W^0 is denoted as the subspaces of W containing functions with normal traces equal to 0.

Let $\mathcal{F}_h$ be the quasi-uniform partition of Ω into rectangles in two dimensions with mesh size h. The lowest-order Raviart–Thomas–Nédélec (RTN) space on rectangles [13,20] is considered. Thus, on an element $D \in \mathcal{F}_h$, we have

$$W_h(D) = \{(\alpha_1 x + \beta_1, \alpha_2 y + \beta_2)^T : \alpha_i, \beta_i \in R, i = 1, 2\},$$
$$S_h(D) = \{\alpha : \alpha \in R\}.$$

Next the standard nodal basis is used, where the nodes are at the centers of the elements for S_h, and the nodes are at the midpoints of edges for W_h. Moreover, the grid points are denoted by

$$(x_{i+1/2}, y_{j+1/2}), i = 0, \ldots, N_x^h, j = 0, \ldots, N_y^h,$$

next the notations similar to those in [21] are used.

$$\begin{aligned}
x_i &= (x_{i-\frac12} + x_{i+\frac12})/2, i = 1, \ldots, N_x^h,\\
h_i^x &= x_{i+\frac12} - x_{i-\frac12}, i = 1, \ldots, N_x^h,\\
h_{i+\frac12}^x &= x_{i+1} - x_i = (h_i^x + h_{i+1}^x)/2, i = 1, \ldots, N_x^h - 1,\\
y_j &= (y_{j-\frac12} + y_{j+\frac12})/2, j = 1, \ldots, N_y^h,\\
h_j^y &= y_{j+\frac12} - y_{j-\frac12}, j = 1, \ldots, N_y^h,
\end{aligned}$$