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A non-autonomous system leading to cyclic chaotic sets to model physiological rhythms

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ABSTRACT

It is proposed a nonlinear system to model highly complex states of rhythms, whose patterns of activity seem irregular. A non-autonomous system which takes into account both exogenous and endogenous influences. The dynamic behaviors of its stroboscopic map are investigated, by using triangular systems. The model provides a theoretical framework for addressing cyclic transitions between chaotic sets. The analysis underlines the role of the parameters in the structure and shape of the attractors, so to be in agreement with experimental data.

1. Introduction

Since a few decades, nonlinear models are used to simulate the phenomena of interest in many fields, not only in physics and engineering, which was the natural framework at the early stage of the development of dynamical systems, but also in biology, chemistry as well as in economics and social sciences. In particular, in the present work we propose a nonlinear system to model highly complex states of rhythms.

Recall that physiological rhythms such as heart beating, neuronal activities, daily cycle of sleep-wake and also the release of hormones are vital phenomena in our life. In response to the fluctuations from the environment and the changes in the endogenous conditions, internal biological mechanisms in the most of the organisms lead to complex fluctuations in physiological rhythms [12,17,36]. Although the dominant frequency of these rhythms is mainly determined by the periods of circadian oscillation which is most likely synchronized with the environmental cycles, the external triggers of events together with internal reactions (e.g., thoughts, emotions, perceptions, etc.) can disturb these rhythms in an unpredictable fashion and results in occurrence of cyclic chaotic oscillations [12,18]. Thus, the patterns seem to have a chaotic behavior.

In addition, numerous lines of evidence in the area of psychiatry suggest that abnormal fluctuations in rhythms for example in sleep/wake pattern could be considered as diagnostic symptoms of major psychiatric disorders like unipolar and bipolar depression as well as schizophrenia [3,18,36,42]. Particularly, at the instance of bipolar disorder, for example, while the dominant frequency of sleep/wake cycles (determined by exogenous cycles) remains approximately constant, changes in the fluctuation patterns reveal much information about adaptation mechanisms in circadian clock in a response to the endogenous conditions [21]. Therefore, as for many natural cyclic phenomena, a mathematical representation for the biological rhythms in terms of deterministic oscillatory models may be proposed, both via continuous and discrete time

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systems. It is worth to mention that in both cases the systems may present cyclic behaviors as well as aperiodic and chaotic behaviors. Several papers deal with models in continuous time systems of biological interest (see e.g. [14,32], and references therein), and in some cases such systems conveniently capture the experimental phenomenology (see for example [15] and references therein).

Here we are mainly interested in discrete dynamical systems, or maps, which are receiving growing attention in the physics of complex systems and processes (see e.g. [5,11,16,20]), each state (a vector of a finite number of variables) is determined by the state in the previous time.

Clearly, a discrete time system can also be obtained by discretization of a continuous time model, or by a suitable Poincaré section, This second approach (a Poincaré section) is just a convenient tool, when possible, which leads to the properties of the continuous time system (see e.g. [39,40]). Differently, the discretization of a continuous time model leads to a map whose dynamic behavior often depends on the used discretization (examples are in [4] and [9]). In such cases, the dynamics may also be quite different from those occurring in the continuous time model. In the applied context, also this can be useful, as to determine which approach is more appropriate in modeling a phenomenon, it will ultimately be the comparison with the experimental results.

Differently, some systems may be directly modeled in terms of discrete relationships among the variables. What is called *a map based modeling approach.* In our case, it seems to be a powerful tool for investigating the inherently recursive process of rhythm generation. Moreover, by using particular non-autonomous systems, as we do, it is possible to take into account the nature of the fluctuations, which are influencing the states.

Accordingly, we propose a mathematical model in discrete time to investigate one of the possible frameworks to design a process to make rhythmic oscillations with chaotic fluctuations. In fact, unpredictable fluctuations related to biological rhythms can be considered as a trajectory moving cyclically through different chaotic sets in the phase space of a system with a multi-band strange attractor. A simple model of a recurrent nonlinear process exhibiting adaptable cyclic chaotic sets can provide the conceptual framework to examine the rhythmic patterns with irregular oscillations. Therefore, we developed a new model which accounts for the fundamental chaotic time series of physiological rhythms through changes in the model's parameters.

The model takes into account that the highly complex states of rhythms can be symbolized through variables which depend on the previous state as well as on discrete exogenous and endogenous influences. Furthermore, in contrast to previous models of rhythm generation, we have chosen to define the state as a vector quantity in the complex plane in which the magnitude of the complex variable stands for intensity of the state, and its angle quantitatively shows its relative phase. As we shall see in the next sections, the model provides a theoretical framework addressing cyclic transitions between pieces of chaotic attracting sets, through which the diverse morphology of the attractors can be captured.

Thus, the goal of the present work is twofold. On one side we need a model which takes into account both exogenous and endogenous influences, on the other side we need to know the relation between the model's parameters and the shape of the cyclic chaotic sets, as evidenced in several different experiments.

The rest of the paper is as follows. In Section 2 we describe the proposed model, which leads to a non-autonomous system. Non-autonomous systems are in general more difficult to study with respect the autonomous ones. However, in our proposal the exogenous influences may be considered with periodic occurrence, so that we can deal with the stroboscopic map which is a three dimensional autonomous system. The several different possibilities as dynamic results are described in Section 3, showing how the parameters in the model influence the outcome. The stroboscopic map is also of a particular structure, as it is triangular. Moreover, it can be decoupled in a pair of companion two-dimensional systems (with very similar dynamics) representing the dynamics of two projections, whose investigation is simplified by the triangular structure of the systems. We assume as driving function the standard logistic map. This leads to a perfect knowledge of the bifurcations occurring in the driving function, and in particular of the chaotic regimes which are of interest. The related structure of cyclical chaotic sets in the three dimensional phase space are investigated mainly via numerical tools. However, these clearly underline the role of the parameters in the structure and shape of the attractors. This role has been evidenced in Section 4, showing the possible simulation of a realistic case. Section 5 concludes.

2. The model

The key idea in the model that we propose is to design a simple non-autonomous linear recurrent system in the state variable z_k, which is a complex variable $z_k = x_k + iy_k$, representing the vector quantity of a physiological variable at time k (e.g. neural activity, secretion of a certain hormone or the level of electrical activity in a syncytium) so that it can be written in the form $z_{k+1} = F(z_k; a_k, b_k)$ as follows:

$$z_{k+1} = a_k z_k + b_k \tag{1}$$

where the slope and the offset are changing dynamically in time. While uncertain environmental influences mainly determine the dynamic pattern of the variation in the slope, by adjusting the dynamics of the offset b_k the system internally responds to these external events. This equation has the goal to propose a biologically reliable rhythm generation formalism. That is, the external influences can be modeled as either noise (with b_k) or chaotic patterns (with a_k). In natural phenomena the pattern generation paradigm has its own rules and is not to be treated as noise. It is a reliable assumption in the area