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Corrigendum to “Basins of attraction for optimal eighth-order methods to find simple roots of nonlinear equations”

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ABSTRACT

In this note we correct the error in the order of convergence of 3 methods we mentioned in the paper by Neta et al. (2014). We will show how the order of convergence is related to the conjugation analysis.

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1. Introduction

In the recent paper [1], the authors analyzed several eighth order methods for obtaining simple roots. We wish to extend and clarify the discussion and to point out a few errors in the paper. In several of the graphs in the above paper, black dots were embedded within the basin of attraction of the given function and iteration method. Within our computer program a black dot denotes that the iteration had not converged after a specified number of iterations. However, in many of the cases, we found, after careful review, the iteration process did converge. All of the algorithms had three stages. The first two stages were well-known second and fourth order algorithms followed by a Newton-like step where either the function or the derivative was replaced by a Hermite interpolating polynomial. This replacement was performed to reduce the overall number of function and derivative evaluations. The iterations were converging so fast that the convergence occurred at an intermediate stage and resulted in an overflow at the last stage. In some cases, convergence occurred after a single iteration of the three stage process. This has been corrected so that convergence is tested at the intermediate stages.

Example 6 of the previous paper was for a polynomial of degree 6 with complex coefficients with zeroes located at $(-1/2-i/2, -3i/2, -1+2i, 1-i, i, 1)$. All of the calculations were computed with the correct equation, but there is a typo in one of the coefficients throughout the paper. The coefficient of the term z should have been $-(i+11)/4$ as opposed to $(i-11)/4$.

Although these algorithms were fast and produced some of the very best basins of attraction, we found, after some further analyses, both analytical and numerical, that the algorithms were not always of eighth order. At times some were only of fourth or sixth order. Several of the algorithms presented in [1] have been eliminated from further study since they were universally the weakest algorithms presented in [1]. In Section 2, we briefly list the algorithms to be studied in this paper. In Section 3, we discuss Möbius transformations and the role they play in the study of iteration algorithms. In Section 4, we discuss the order of convergence of the methods. Numerical results will be presented in Section 5 and Conclusions are presented in Section 6.

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2. Descriptions of algorithms

In this section, we list the 4 methods to be discussed.

-I- The first method, denoted JHID,

$$\begin{aligned} y_n &= x_n - \frac{2}{3}u_n \\ t_n &= x_n - \frac{1}{2}u_n - \frac{1}{2}\frac{u_n}{1+\frac{3}{2}\left(\frac{f'(y_n)}{f'_n}-1\right)} \\ x_{n+1} &= t_n - \frac{f(t_n)}{H'_3(t_n)} \end{aligned} \tag{1}$$

where

$$u_n = \frac{f_n}{f'_n}, \tag{2}$$

$f_n = f(x_n),\ f'_n = f'(x_n),$

$$H'_3(t_n) = 2(f[x_n,t_n] - f[x_n,y_n]) + f[y_n,t_n] + \frac{y_n - t_n}{y_n - x_n}(f[x_n,y_n] - f'_n), \tag{3}$$

and the divided differences $f[a,b]=\frac{f(b)-f(a)}{b-a}$.

-II- The second method, denoted JHIF, is similar to the first scheme except we replaced the function in the last sub-step by the Hermite polynomial instead of replacing the derivative.

$$\begin{aligned} y_n &= x_n - \frac{2}{3}u_n \\ t_n &= x_n - \frac{1}{2}u_n - \frac{1}{2}\frac{u_n}{1+\frac{3}{2}\left(\frac{f'(y_n)}{f'_n}-1\right)} \\ x_{n+1} &= t_n - \frac{H_3(t_n)}{f'(t_n)}, \end{aligned} \tag{4}$$

where

$$H_3(t_n) = f_n + f'_n\frac{(t_n-y_n)^2(t_n-x_n)}{(y_n-x_n)(x_n+2y_n-3t_n)} + f'(t_n)\frac{(t_n-y_n)(x_n-t_n)}{x_n+2y_n-3t_n} - f[x_n,y_n]\frac{(t_n-x_n)^3}{(y_n-x_n)(x_n+2y_n-3t_n)}. \tag{5}$$

-III- The third scheme considered is due to Wang and Liu [2]. Here we have the original eighth order method denoted by WL

$$\begin{aligned} y_n &= x_n - u_n \\ t_n &= y_n - \frac{f(y_n)}{f'_n}\frac{f_n}{f_n - 2f(y_n)} \\ x_{n+1} &= t_n - \frac{f(t_n)}{H'_3(t_n)}, \end{aligned} \tag{6}$$

where $H'_3(t_n)$ is defined by (3). Note that the first two substeps are Ostrowski's method.

-IV- The last scheme, denoted WLN, is similar to the third scheme except we replaced the function in the last sub-step by the Hermite polynomial instead of replacing the derivative.

$$\begin{aligned} y_n &= x_n - u_n \\ t_n &= y_n - \frac{f(y_n)}{f'_n}\frac{f_n}{f_n - 2f(y_n)} \\ x_{n+1} &= t_n - \frac{H_3(t_n)}{f'(t_n)}, \end{aligned} \tag{7}$$

where $H_3(t_n)$ is given by (5).

Note that all these methods can be written as

$$x_{n+1} = x_n - u_n H_f(x_n, y_n, t_n),$$

where H_f are given in Table 1.