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Generalized multi-synchronization viewed as a multi-agent leader-following consensus problem

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ABSTRACT

The work presents the generalized multi-synchronization of strictly different chaotic systems problem as a multi-agent leader-following consensus problem with fixed and not strongly connected topology from a differential algebra point of view. This problem is solved by using the differential primitive element as a linear combination of state measurements and control inputs from which it is possible to construct a family of transformations to carry out the multi-agent systems to a Generalized Observability Canonical Form Multi-output (GOCFM). Moreover, a dynamic consensus protocol is designed such that the states of the followers asymptotically converge to the state of the leader.

1. Introduction

In general, a group composed of multiple interacting dynamical systems (agents) is referred to as Multi-Agent Systems (MAS). It is required that from this interaction, agents coordinate their behavior or agree upon some variable of interest. In this framework, consensus means that MAS have reached an agreement on a variable of interest that depends on the state of all agents due to a specified interaction algorithm for information exchange known as consensus protocol. The main objective of this protocol is to impose similar dynamics on the states of each agent. Consensus problems have a long history in computer science, they are considered the basis of distributed computing [1]. The ideas behind it have been extended to numerous applications such as rendezvous, formation control, flocking, attitude alignment, sensor networks and synchronization [2–5]. In the synchronization problem, the master–slave topology used for exchange of information is unidirectional type and fixed in the sense that a master system passes information to a slave system. This is an interesting topic that can be treated as a consensus problem. In this case we say that all agent systems achieve the consensus if all nodes in the network approach to a common trajectory when $t \to \infty$.

The synchronization of chaotic systems has been extensively researched since its introduction by Pecora and Carrol [6]. In literature, there exist several types of chaos synchronization: complete synchronization (CS), generalized synchronization (GS), phase synchronization and lag synchronization. Here, we treat the problem of generalized multi-synchronization (GMS) that appears naturally in a master–slave topology [7,8]. It is worth mentioning that when the dynamics of MAS are strictly different, the GMS problem is harder to solve than when the dynamics are identical (complete synchronization). From a control point of view, the synchronization problem is given in terms of a synchronization manifold which depends on states or outputs of the system. Finding a synchronization manifold is not a simple task when the MAS are composed of strictly

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different agents, this manifold is not trivial or does not necessarily exist. Therefore, it is unclear whether agent systems with different dynamics can be synchronized. However, in our GMS problem there exists a synchronization manifold (algebraic manifold) M_l defined in the sense of Definition 7 for strictly different systems that can be explicitly given. In this paper, our aim is to work with synchronization of strictly different chaotic systems as a leader-following consensus problem by using differential algebra techniques. Since, we treat the GMS problem in a master–slave topology, we consider a class of nonlinear systems as MAS and a directed spanning tree to model the unidirectional interaction in the network. It is natural that no connections are available between the followers because of the topology. The main contribution of this work is to show that the leader and all followers are in a state of GMS when the leader-following consensus of agent systems is achieved, that is to say, GMS is a leader-following consensus problem. To achieve this task a dynamic consensus protocol is designed. The rest of the paper is organized as follows: In Section 2 we establish basic concepts about differential algebra, consensus and generalized synchronization, in Section 3 we present the problem formulation, in Section 4 we show the chaotic agent systems used in a numerical example where the methodology proposed is applied on a network with one leader (Colppits system) and two followers (Rössler and Chua system). Finally, in Section 5, we give some concluding remarks.

2. Preliminaries

2.1. Differential algebra

Definition 1. A differential field extension L/k is given by two differential fields k, L such that:

1. k is a subfield of L,
2. The derivation of k is the restriction of k due to derivation of L.

Definition 2. An element $a \in L$ is said to be differentially algebraic over k if and only if it satisfies a differential equation $P(a, \dot{a}, \dots, a^{(\alpha)}) = 0$, where P is a polynomial over k in a and its time derivatives, if a does not satisfy such differential equation we say that the element a is differentially transcendental over k.

We consider the definition about differential primitive element that states there exists a single element $\delta \in L$ such that $L = k\langle\delta\rangle$, i.e. L is differentially generated by k and δ, where k is the differential field and δ is the differential primitive element [9].

Definition 3. A dynamics is defined as a finitely generated differential algebraic extension $L/k\langle u\rangle$ of the differential field $k\langle u\rangle$, where $k\langle u\rangle$ denotes the differential field generated by k and a finite set of differential quantities $u = (u_1, u_2, \dots, u_m)$.

Definition 4. A system's family is Picard–Vessiot (PV) if and only if the $k_{\bar{i}}\langle u\rangle$ vector space generated by the derivatives of the family $\{\bar{y}_{\bar{i}}^{(n_{\bar{i}})}, n_{\bar{i}} \geq 0, 1 \leq \bar{i} \leq p\}$ have finite dimension, where $\bar{y}_{\bar{i}}$ is the $\bar{i}th$ differential primitive element of the family of differential primitive elements.

2.2. Algebraic graph theory and consensus

To model the information flow (interaction) between "$r+1$" agents consider a directed graph $G = (V, E, A)$ with a set of nodes $V = \{0, 1, \dots, r\}$, a set of edges $E \subseteq V \times V$, and an adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{(r+1)\times(r+1)}$ with nonnegative adjacency elements a_{ij} defined as

$$a_{ij} = \begin{cases} 1 & (j, i) \in E \\ 0 & \text{otherwise} \end{cases}$$

Let $L = [l_{ij}] \in \mathbb{R}^{(r+1)\times(r+1)}$ be the nonsymmetrical graph Laplacian matrix induced by the information flow G defined by

$$l_{ij} = \begin{cases} \sum_{k=1,\, k\neq i}^{r+1} a_{ik} & j = i \\ -a_{ij} & \text{otherwise} \end{cases}$$

Consider each node of G as a dynamic agent with nonlinear dynamics. The MAS dynamics is given by

$$\begin{aligned} \dot{x}_{i1} &= f_{i1}^T(x_{i1}, x_{i2}, \dots, x_{in}, u_{i1}, u_{i2}, \dots, u_{in}) \\ \dot{x}_{i2} &= f_{i2}^T(x_{i1}, x_{i2}, \dots, x_{in}, u_{i1}, u_{i2}, \dots, u_{in}) \\ &\vdots \\ \dot{x}_{in} &= f_{in}^T(x_{i1}, x_{i2}, \dots, x_{in}, u_{i1}, u_{i2}, \dots, u_{in}) \end{aligned} \tag{1}$$

for all $i \in V$, where $x_{i1} = (x_{01}, x_{11}, \dots, x_{r1})$, $x_{i2} = (x_{02}, x_{12}, \dots, x_{r2}), \dots$, $x_{in} = (x_{0n}, x_{1n}, \dots, x_{rn}) \in \mathbb{R}^{r+1}$, $u_{i1} = (u_{01}, \dots, u_{r1})$, $\dots$, $u_{in} = (u_{0n}, \dots, u_{rn}) \in \mathbb{R}^{r+1}$.

Let x_{ij} denote the value of node j of the ith agent system for all i, j fixed. In this particular case, assume all interactions for nodes $x_{i1}, x_{i2}, \dots, x_{in}$ have the same information flow G (i.e. they have the same Laplacian matrix) for $i \in V$ fixed. Thus, we