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Continuous global optimization through the generation of parametric curves

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ABSTRACT

In this paper we develop a new approach for solving a large class of global optimization problems. The objective function is only continuous, non-smooth and non-Lipschitzian, defined on a rectangle of $\mathbb{R}^n$. This approach is based on the generation, in the feasible set, of a family of parametrized curves satisfying certain properties combined with the one-dimensional Evtushenko algorithm. To accelerate the corresponding mixed algorithm, we have incorporated in a variant a Pattern Search-type deterministic local optimization method and in another variant a new stochastic local optimization method. Both variants have been applied to several typical test problems. A comparison with some well known methods is highlighted through numerical experiments.

1. Introduction

Let f be a continuous function on the rectangle $X = \prod_{i=1}^{n} [a_i, b_i] \subset \mathbb{R}^n$, where a_i, b_i are real numbers for $i = 1, \ldots, n$. In this work we consider the global optimization problem of looking for the value f^* such that

$$f^* = \min_{x \in X} f(x). \qquad (P)$$

Many scientific and engineering problems may be formulated as optimization problems that involve objective functions which are only continuous and do not possess strong mathematical properties (such as convexity, differentiability, Lipschitz continuity etc.) [17,18,45,51]. In the last three decades an assortment of deterministic and random search methods have been developed to deal with these problems. The monographs [13,17,33] give a good overview of methods and software for continuous global optimization. In particular, the DIRECT methods (DIviding RECtangles algorithms) [11,12,19,20] and the evolutionary methods [5,39,40,48] are largely used and constantly improved. This has allowed to considerably reduce the number of evaluation points of f and the calculation time that is necessary to obtain an approximate solution. The deterministic methods, as is well known, guarantee asymptotic convergence to the global minimum whereas random search methods ensure convergence in probability. However, without additional assumptions concerning problem (P), the theoretical convergence results do not supply a stopping criterion (for the algorithm) which guarantees the global minimum with a prescribed accuracy $\varepsilon > 0$ in a finite number of iterations [17,33,45,51]. We suggest here an algorithm which can be

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considered as a relaxation of the reducing transformation method which is studied by numerous authors, see e.g. Butz [6], Strongin [41], Strongin and Sergeyev [42], Cherruault and Guillez [7]. Recall that it consists in reducing a multivariate problem to a univariate one by using a space filling curve to approximate the feasible domain. The curves used belong to two different classes. Sergeyev and his collaborators have systematically improved upon, over the years, the reducing transformation method by considering the approximations of Peano-type curves. The monograph [37] examines several theoretical and numerical aspects of this method. Cherruault and Mora have, on the other hand, used another class of curves called α-dense [8]. A curve in X is α-dense if simply all points in X are within a distance at most $\alpha\,(\alpha > 0)$ from the curve. Of course, the approximations of Peano curves are also α-dense but the curves used in the Cherruault method do not converge towards Peano-type curves. Though both types of curves have the common property of approximating at will all the points of the feasible set X, they do differ concerning certain aspects such as smoothness, uniformity and asymptotic behavior. The reducing transformation coupled with certain one-dimensional optimization methods has proved to be efficient for solving various types of global optimization problems where the objective function is Lipschitzian (or Hölderian) and the feasible set is relatively small [15,35,37,44]. But it is less efficient if the feasible domain is large enough or of dimension larger than 4 since the algorithm produces an excessive number of evaluation points of f. One reason for this, is that the use of a single curve which closely approximates the feasible domain in the same way all along its wanderings lacks overall flexibility. Our idea is then to work on a certain number of curves that suitably scan X and allow more freedom while exploring the regions of X combined with two optimization techniques: a univariate global optimization method and a multivariate local search one. In the early phases of the algorithm, the curves are generated with a densifying parameter α, which is relatively large, but they are sufficiently spread to cover a vast region; then the parameter α progressively decreases as the algorithm evolves. Due to the role played by the curves in the proposed method, we thought it is convenient to use the curves in the Alienor reducing transformation. That way, the proposed method is composed of two distinct search procedures which work together: an exploration procedure whereby parametrized curves are generated and coupled with the one dimensional algorithm of Evtushenko and an exploitation procedure imparted to a local optimization. While scanning the feasible set X, the exploration phase, which will be designed as Alienor–Evtushenko, searches for new solutions or spots an attraction zone of a new local minimizer. The exploitation phase for its part, uses the results of the exploration phase to select the most promising sub-region of X and tries to head locally for a point which does improve the value of f. The last solution thus obtained gives a new opportunity to accelerate the exploration procedure. Two variants of our approach have been applied to the problem (P). In a variant, we use in the exploitation procedure the well known deterministic local optimization method of Generalized Pattern Search [3,21]. However, for objective functions that are apt to exhibit violent variations with a large number of local minima, it is rather recommended to call upon a stochastic procedure. Thus, in another variant, we introduce in the exploitation phase a new stochastic local search method. The latter is inspired by Hooke and Jeeves Pattern Search method [16]. The one dimensional Evtushenko algorithm is well adapted to this approach since the record obtained on a curve can be exploited by this algorithm to quicken the search for the global minimum over another curve. This later procedure cannot be realized with other covering methods such as the different extensions or modifications of the algorithm of Pyavskii–Shubert [9,18,33], despite their efficiency.

Let us now consider a key result and recall that if a function g is evaluated at the points $x_1, x_2, \dots, x_k$, we call the value $g_k^* = \min\{g(x_1), g(x_2), \dots, g(x_k)\}$ a record (at the iteration k) and a point x_k^* where $g(x_k^*) = g_k^*$ a record point. Our approach is essentially based on the following result

Theorem 1. *A function f defined on a compact set X of $\mathbb{R}^n$ is continuous if and only if $\forall\,\varepsilon > 0$ there exists a constant $K > 0$ such that $\forall\,x, y \in X$*

$$|f(x) - f(y)| \le K\,\|x - y\| + \varepsilon. \tag{1}$$

For the proof one can see [35] and the idea is that the space $L(X, \mathbb{R})$ of Lipschitz functions on X, i.e. the set of functions f such that $\exists\,K > 0$ for which $|f(x) - f(y)| \le K\,\|x - y\|$ for all $x, y \in X$, is dense in the space $C(X, \mathbb{R})$ of continuous functions on X endowed with the sup-norm $\|f - g\| = \sup_{x\in X} |f(x) - g(x)|$. Another version of this theorem is in [47] where it is assumed, however, that X is convex (which is not necessary if X is compact). A drawback of the above theorem is that the constant K in (1) is unknown in general (had it be known, then with slight modification, all global optimization methods used when f is Lipschitzian carry over to this case). In practice, there exist a variety of techniques allowing for estimating K [37,42,43]. The annoying thing is that many auxiliary calculations are needed even though the solution is not guaranteed in a finite number of evaluation points. The idea in our approach is to construct a suitable strictly increasing sequence of real numbers (K_j) tending to infinity, with first term $K_1 > 0$, such that [1] for $\varepsilon > 0$, $\exists\,j_0 \in \{1, 2, \dots\}$ satisfying $|f(x) - f(y)| \le K_{j_0}\|x - y\| + \varepsilon/4$, $\forall x, y \in X$. The constant K_{j_0} guaranties the convergence of the algorithm towards its global minimum in a finite number of iterations (within the prescribed accuracy $\varepsilon > 0$). The proposed method performs a series of applications of the coupled Alienor–Evtushenko algorithm by changing successively the parameters K_j, $j = 1, 2, \dots$. At the step j, the Alienor method generates a simple curve (without double points) well spread over X and α_j-dense with parameter α_j depending on K_j. Then the Evtushenko algorithm, which has a variable step-length that also depends on the constant K_j and on the record obtained during the $(j-1)$ preceding steps, improves the value of the objective function on this curve. The local

[1] For practical reasons, we take $\varepsilon/4$ instead of ε (see the proof of Theorem 3 of Section 3.4).