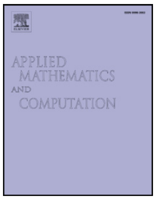
Applied Mathematics and Computation 282 (2016) 117–127

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc



Iterated symmetric three-player prisoner's dilemma game

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ARTICLE INFO

MSC:
91A06
91A10
91A15
91A20

Keywords:
Iterated games
Prisoner's dilemma
Payoff matrix
Symmetric games
Tit For Tat strategy
Evolutionary games

ABSTRACT

Although, most game theory researches on the prisoner's dilemma have centered on two-player models, it is possible to create it to be consisted of three or even more players. In this paper, we are interested in the model of three-player iterated prisoner's dilemma game where, each player has two choices. The action of each strategy in this model depends on the previous action of the last round. Each strategy is presented by finite state of automata. We used a computer program to calculate the payoff values resulting from the actions of all possible strategies. We study the behavior of four different strategies related to Tit for Tat concept. The conditions of each strategy to be the best are determined. In Appendix section, we design an algorithm and implement it using the Java programing language to facilitate the calculations.

1. Introduction

The prisoner's dilemma (PD) is a traditional game model for the study of decision-making and self-interest [1,2]. It is only one of many illustrative examples of the logical reasoning and complex decisions involved in game theory. The mechanisms that drive the (PD) are the same as those that are faced by marketers, military strategists, poker players, and many other types of competitors [3–5]. This dilemma can multiply into hundreds of other more complex dilemmas. The dilemma has widely been addressed in different disciplines such as artificial intelligence, economics [6,7], biology [8], physics, networks [9], business [10], mathematics [11,12], philosophy, public health, ecology [13], traffic engineering [14], sociology and computer science [15].

In the prisoner's dilemma, two players are faced with a choice, they can either cooperate or defect. Each player is awarded points (called payoff) depending on the choice they made compared to the choice of the opponent. Each player's decision must be made without knowledge of the other player's next move. Prior agreement between the players concerning the game is not allowed. If both players cooperate they both receive a reward, R. If both players defect they both receive a punishment, P. If one player defects and the other cooperate, the defector receives a reward, T the temptation to defect, while the player who cooperated is punished with the sucker's payoff, S [16]. We can represent the payoff matrix as the following:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \tag{1}$$

Where, $T > R > P > S$ should be satisfied [17].

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http://dx.doi.org/10.1016/j.amc.2016.02.009
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In single game of prisoner's dilemma (known as one shot), defection is the dominant strategy (the Nash equilibrium) [18]. Thus both players will defect earning rewards of P points rather than the R points that mutual cooperation could have yielded. The iterated prisoner's dilemma (*IPD*) is an interesting variant of (*PD*) where, the dominant mutual defection strategy relies on the fact that it is a one shot game with no future while, the key of the (IPD) is that the two players may meet each other again, and develop their strategies based on the previous game interactions [19]. Therefore a player's move now may affect how his/her opponent behaves in the future and thus affect the player's future payoffs, and this removes the single dominant strategy of mutual defection because, the players will use more complex strategies which depend on the game history to maximize the payoffs that they receive. In fact, under the correct circumstances mutual cooperation can emerge [10,20].

Wang et al. have studied the evolution of public cooperation on two interdependent networks that are connected by means of a utility function, which determines to what extent payoffs in one network influence the success of players in the other network [9,21]. Also, they have shown that the percolation threshold of an interaction graph constitutes the optimal population density for the evolution of public cooperation, and they have demonstrated this by presenting outcomes of the public goods game on the square lattice with and without an extended imitation range, as well as on the triangular lattice [22–24]. More importantly, they have found that for cooperation to be optimally promoted, the interdependence should stem only from an intermediate fraction of links connecting the two networks, and that those links should affect the utility of players significantly [25]. Recently, they have studied the evolution of cooperation in the public goods game on interdependent networks. They have shown that, increasing the relevance of the average payoff of nearest neighbors on the expense of individual payoffs in the evaluation of utility increases the survivability of cooperators [26,27]. They have showed that the interdependence between networks self-organizes so as to yield optimal conditions for the evolution of cooperation [28].

Perc and Szolonki have worked on studying the enhancement of cooperation, and the impact of diverse activity patterns on the evolution of cooperation in evolutionary social dilemmas [29–34]. They have showed that spatially and temporally white additive Gaussian noise introduced in the payoff matrix of an evolutionary spatial prisoner's dilemma game can facilitate and maintain cooperation in a resonant manner depending on the level of random variations [35]. Moreover, they have showed that extortion is evolutionary stable in structured populations if the strategy updating is governed by a myopic best response rule [36,37]. While, Xia et al. have focused on the weak prisoner's dilemma on random and scale-free (SF) networks, and have shown that degree-uncorrelated activity patterns on scale-free networks significantly impair the evolution of cooperation, and they have studied how the heterogeneous coupling strength affects the evolution of cooperation in the prisoner's dilemma game with two types of coupling schemes (symmetric and asymmetric ones) [38]. Their results convincingly demonstrated that the emergence or persistence of cooperation within many real-world systems can be accounted for by the interdependency between meta-populations or sub-systems [39]. Moreover, they have put forward an improved traveler's dilemma game model on two coupled lattices to investigate the effect of coupling effect on the evolution of cooperation [40]. Their results are surprisingly conducive to understanding the cooperation behavior of traveler's dilemma game within many real world systems, especially for coupled and interdependent networked systems [41].

Game theory has been extended into evolutionary biology, which has generated great insight into the evolution of strategies under both biological and cultural evolution. The replicator equation, which consists of sets of differential equations describing how the strategies of a population evolve over time under selective pressures, has also been used to study learning in various scenarios [42–44]. There are various approaches to construct dynamics in repeated games [45–47]. Kleimenov and Schneider have proposed approach of constructing dynamics in the repeated three-person game to give a tool for solving various optimization problems, for example, the problem of minimizing time of using abnormal behavior types. In their approach, two players act in the class of mixed strategies and the third player acts in the class of pure strategies [48,49]. Matsushima and Ikegami have discussed the similarity between a noisy $2p-IPD$ and a noiseless $3p-IPD$ game where the role of noise in the two-person game is replaced by the third player in the three-person game. It is known that, due to the noise, Tit for Tat loses its robustness and is taken over by more complex strategies in a noiseless *IPD* game, but in the $3p-IPD$ game, even without noise, tit for tat loses its robustness and is also taken over by more complex strategies. They found that similar strategies take over tit for tat in both situations. It is also found that game strategies in an automaton form can be understood as a combination of defensive and offensive substructures. A recognition of these substructures enabled them to study the mechanism of robustness in the strategies of the $3p-IPD$ game [50].

The existence and implications of alternative stable states in ecological systems have been investigated extensively within deterministic models. Sun et al. have studied the role of noise on the pattern formation of a spatial predator-prey model with Allee effect, and have showed that the spatially extended system exhibits rich dynamic behavior. More specifically, the stationary pattern can be induced to be a stable target wave when the noise intensity is small. As the noise intensity increases, patchy invasion emerges. Their results indicate that the dynamic behavior of predator–prey models may be partly due to stochastic factors instead of deterministic factors, which may also help to understand the effects arising from the undeniable susceptibility to random fluctuations of real ecosystems [51]. Also, they have presented a spatial version of the predator–prey model with *HollingIII* functional response, which includes some important factors such as external periodic forces, random fluctuations, and diffusion processes. They found that, noise can lead the spiral waves to be chaotic patterns [52]. Additionally, they have presented a numerical evidence of complicated phenomenon controlled by noise in a spatial epidemic model, where the number of the spot decreases as the noise intensity being increased. Their results showed that noise plays an important role in the pattern formation of the epidemic model, which may provide guidance to prevent and