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Disturbance rejection in nonlinear systems based on equivalent-input-disturbance approach

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ABSTRACT

This paper presents a new system configuration and a design method that improves disturbance rejection performance for a nonlinear system. The equivalent-input-disturbance (EID) approach is used to construct an EID estimator that estimates the influence of exogenous disturbances and nonlinearities on the output of the system. Sufficient stability conditions for state- and output-feedback control are derived in terms of linear matrix inequalities. New EID-based control laws that combines an EID estimate with a state- or an output-feedback control laws ensure good control performance. A numerical example illustrates the design method. A comparison between the EID-based control, the conventional disturbance observer, the disturbance-observer-based-control, and the sliding mode control methods demonstrates the validity and superiority of the EID-based control method.

1. Introduction

Nonlinearities appear in spacecraft, robots, industrial processes, biological phenomena, power systems, and many other systems. Since a dynamic system is also often affected by exogenous disturbances, a control system is required to have the ability to handle both nonlinearities and disturbances to provide satisfactory control performance.

A number of design methods for rejecting disturbances in nonlinear systems have been proposed. For example, a method based on the internal-model principle solved the problem of rejecting an arbitrary disturbance by employing an internal model of the disturbance [1–3]. A successive approximation algorithm [4,5] and the disturbance-observer-based control (DOBC) method [6,7] have been used to reject known exogenous disturbances. An adaptive control method was devised to reject a disturbance generated by an unknown linear exosystem with known order and distinct eigenvalues [8,9]. Sliding mode control (SMC) [10,11] is an effective method to reject disturbances. However, there is a trade-off between the performance of disturbance rejection and others, and chattering exists in the system. However, besides SMC method, methods based on a disturbance model may require a disturbance compensator with a very high order to handle a complicated disturbance. Moreover, most of those methods require a priori information on the disturbance. This may not be possible in many cases of control engineering practice.

Various methods have also been devised to deal with system nonlinearities so as to improve control performance. For example, the generalized extended-state observer method was developed to handle nonlinearities that can be treated as

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step-type disturbances [12]. The small-disturbance theory yields a linearized model of a nonlinear system at an equilibrium point. However, since it ignores high-order nonlinear terms, it is unsuitable when operating conditions vary over a wide range [13]. A diffeomorphism has been used to transform a nonlinear system to a quasilinear one [14]. And the expansion of a Maclaurin series has been applied to a nonlinearity to yield a linear approximate model [15]. While all these methods create a linear model of a nonlinear system, the model may not be able to describe the true nature of the system when high-order nonlinear terms cannot be ignored. Other methods in addition to linearization methods have been proposed that directly deal with a nonlinear system. For example, neutral-network control [16] and fuzzy control [17], which employ a neural network or a fuzzy logic system to approximate the nonlinearities of a system. They provide satisfactory approximation accuracy, but usually at a very high computational expense.

An alternative to all the methods mentioned above is the equivalent-input-disturbance (EID) approach. An EID is a signal on the control input channel that produces the same effect on the output as actual disturbances do [18]. The method can reject both matched and unmatched disturbances [18–20]. Unlike the conventional disturbance-observer (DOB) method [21], it does not require an inverse model of the plant or prior information on the disturbances [19]. It avoids the cancellation of unstable poles and zeros. And it yields satisfactory disturbance-rejection performance. Unlike other methods, the system configuration is simple. Moreover, taking a nonlinearity as a state-depend disturbance, it can simultaneously handle both disturbances and nonlinearities. An attempt to use the EID approach to compensate the nonlinearity of a two-link gymnast robot for trajectory tracking control showed the validity of using this approach for nonlinearity compensation [20].

This paper presents a method that employs the EID approach for disturbance rejection in a nonlinear system. One advantage of the method is that it rejects disturbances and compensates for nonlinearities without using a transformation of the original nonlinear system. Sufficient conditions of robust stability of the closed-loop system for unknown nonlinearities are devised for state- and output feedback control in terms of linear matrix inequalities (LMIs), and the controllers are designed based on the LMIs. The validity of the method is demonstrated through simulations and through a comparison with the conventional DOB, the DOBC, and the SMC methods.

In the rest of the paper, $\begin{bmatrix} A & B \\ B^{\mathrm{T}} & C \end{bmatrix}$ is indicated by $\begin{bmatrix} A & B \\ \star & C \end{bmatrix}$, $\lambda_m(A)$ denotes the minimum eigenvalue of the matrix A, $\lambda_M(A)$ denotes the maximum eigenvalue of the matrix A, and $\|\cdot\|$ stands for the Euclidean norm.

2. Construction of control system

Consider a nonlinear plant

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + f(x(t)) + B_d d(t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) \in \mathbb{R}^q$ is the output; $d(t) \in \mathbb{R}^{n_d}$ is a disturbance; A, B, B_d, and C are constant matrices with suitable dimensions; and $f(x(t))$ is an unknown nonlinearity.

The following assumptions are made for f and d.

Assumption 1. $f(x(t))$ is an unknown nonlinearity satisfying

$$f^{\mathrm{T}} f \le x^{\mathrm{T}} H_1 x, \tag{2}$$

where H_1 is a given positive definite matrix.

Assumption 2. $d(t)$ is a disturbance satisfying

$$\|d(t)\|_\infty < d_M, \tag{3}$$

where d_M is an unknown positive real number, and $\|\cdot\|_\infty$ is the infinity norm.

The EID-based nonlinear control system (EBNCS) developed in this study contains four parts: the plant, a state observer, an EID estimator, and state feedback (Fig. 1, [19]). When the state of the plant is available (Case 1), the state feedback uses it directly; and when it is unavailable (Case 2), it uses the state reproduced by the state observer.

Since an unknown nonlinearity and a disturbance can be treated as a state-dependent EID, letting the EID be $d_e(t)$ allows us to describe the plant as

$$\begin{cases} \dot{x} = Ax(t) + Bu(t) + Bd_e(t), \\ y(t) = Cx(t). \end{cases} \tag{4}$$

A full-order observer is used to estimate the EID. The state-space representation of the observer is

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu_f(t) + L[y(t) - \hat{y}(t)], \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{5}$$

where $\hat{x}(t)$ is the reconstructed state of $x(t)$.

Let

$$B^+ := (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}, \tag{6}$$