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A class of global fractional-order projective dynamical systems involving set-valued perturbations☆

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ABSTRACT

This paper studies a class of global fractional-order projective dynamical systems involving set-valued perturbations in real separable Hilbert spaces. We prove that the set of solutions for this type of systems is nonempty and closed under some suitable conditions. Furthermore, we show that the set of solutions is continuous with respect to initial value in the sense of Hausdorff metric. Finally, an interesting numerical example is given to illustrate the validity of the main theorem presented in this paper.

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1. Introduction

The connection between a projective dynamical system and an associated variational inequality was revealed in 1993, by Dupuis and Nagurney [8] who proposed the following local projective dynamical system

$$\frac{dx}{dt} = \lim_{\rho\to 0} \frac{P_K(x-\rho N(x)) - x}{\rho}.$$

Later Friesze et al. [10] extended this type of systems to global ones and applied them in solving problems in traffic network equilibrium analysis. Furthermore, they showed that the tatonnement model of a certain traffic problem can be formulated as a simultaneous projective dynamical systems

$$\begin{cases} \dfrac{dh(t)}{dt} = \eta\{P_K(h(t) - \rho ETC(h(t), u(t))) - h(t)\}, \quad \forall t \in [0, T], \\ h(0) = h_0, \\ \dfrac{du(t)}{dt} = \kappa\{P_K(u(t) + \lambda ETD(u(t), h(t))) - u(t)\}, \quad \forall t \in [0, T], \\ u(0) = u_0, \end{cases}$$

where $\rho, \lambda \in R^1_+ = [0, +\infty)$.

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During the past decades, projective dynamical systems have continuously been a hot area in the research fields. Both in-depth theories and wide applications of the projective dynamical systems have been studied extensively. For examples, we refer to [1,4,7,8,10–13,25,28–33,37–39,41–45] and the references therein.

On the other hand, we all know that in the practical world, many real life scientific or engineering problems are neither linear nor nonlinear, which makes it inappropriate to model them with differential systems of integer order. Instead, fractional derivatives provide a better tool than integer order ones to describe many physical processes, especially those with memory and hereditary properties, see [20,23,27,40]. Recently, we noticed that some excellent work relating fractional order differential systems have already appeared in this field (see, for example, [2,5,9,15,21,26]). In particular, In 1984, Torvik and Bagley [27] showed that the fractional order model was effective in describing the behavior of real material. In 2011, Li and Zhang [20] did a survey on the stability of fractional differential equations. In 2012, Ozalp and Koca [23] introduced a fractional order dynamical model of interpersonal relationships. In 2014, Yu et al. [40] studied projective synchronization for fractional neural network. In 2015, Buyukkilic et al. [2] investigated cumulative growth of a physical quantity via Fibonacci method and fractional calculus.

Moreover, in 2014, Wu and Zou [34], for the first time, proposed the following global fractional-order projective dynamical systems in R^n

$$\begin{cases} {}_0^C D_t^{\alpha} x(t) = P_K(x(t) - \rho M x(t) - \rho b) - x(t), \quad t \geq 0, \\ x_i(0) = x_{i0}, \quad i = 1, 2, \ldots, n, \end{cases} \tag{1.1}$$

where $0 < \alpha < 1$, M is a real $n \times n$ matrix. Then the existence and uniqueness of the solution and the existence of the equilibrium point were obtained, and they showed that the equilibrium point was α-exponentially stable. In addition, they gave a numerical algorithm and an example.

Recently, Wu et al. [35] considered a class of fractional set-valued projected dynamical systems in R^n as follows

$$\begin{cases} {}_0^C D_t^{\alpha} x(t) \in P_K(g(x(t)) - \lambda N(x(t))) - g(x(t)), \quad \text{for } t \in [0, h]; \\ x(0) = a, \ x(h) = b, \quad a, b \neq 0, \end{cases} \tag{1.2}$$

where $\alpha \in [1, 2)$, $N : \mathbb{R}^n \to 2^{\mathbb{R}^n}$ is a set-valued mapping. The nonempty and closed of the solutions set was obtained and they showed that the set of solutions was continuous with respect to boundary value in the sense of Hausdorff metric.

Very recently, based on Wardropian user equilibrium tatonnement model, Wu et al. [36] introduced and investigated a system of fractional-order interval projective dynamical systems in $R^n \times R^m$ as follows

$$\begin{cases} {}_0^C D_t^{\alpha} x(t) = P_{K_1}[x(t) - \rho(Ax(t) + A^* y(t)) - \rho a] - x(t), \ t \geq 0, \\ x(0) = x_0, \\ {}_0^C D_t^{\alpha} y(t) = P_{K_2}[y(t) - \lambda(By(t) + B^* x(t)) - \lambda b] - y(t), \ t \geq 0, \\ y(0) = y_0, \end{cases} \tag{1.3}$$

where $0 < \alpha \leq 1$ and

$$\begin{cases} A \in A_I = \left\{ \left(a_{ij}\right)_{n\times n} : \underline{A} \leq A \leq \overline{A}, \quad i.e., \underline{a}_{ij} \leq a_{ij} \leq \overline{a}_{ij} \right\}, \\ A^* \in A_I^* = \left\{ \left(a_{ij}^*\right)_{n\times m} : \underline{A^*} \leq A^* \leq \overline{A^*}, \quad i.e., \underline{a^*}_{ij} \leq a_{ij}^* \leq \overline{a^*}_{ij} \right\}, \\ B \in B_I = \left\{ \left(b_{ij}\right)_{m\times m} : \underline{B} \leq B \leq \overline{B}, \quad i.e., \underline{b}_{ij} \leq b_{ij} \leq \overline{b}_{ij} \right\}, \\ B^* \in B_I^* = \left\{ \left(b_{ij}^*\right)_{m\times n} : \underline{B^*} \leq B^* \leq \overline{B^*}, \ i.e., \underline{b^*}_{ij} \leq b_{ij}^* \leq \overline{b^*}_{ij} \right\}. \end{cases}$$

Then they proved the existence and uniqueness of the equilibrium point under some suitable conditions. Moreover, α-exponential stability of this type of projective dynamical systems is obtained, in addition, two numerical examples was provided.

But to our best knowledge, for the global fractional-order projective dynamical systems in abstract spaces, there are very few results in the existing literatures. The motivation of the present work is to make an attempt in this direction. Moreover, the system may appear perturbation by external factor. In particular, suppose that the perturbation is a set-valued mapping. Inspired and motivated by previous facts, in this paper, we consider the following fractional projective dynamical systems with set-valued perturbations in real separable Hilbert spaces:

$$\begin{cases} {}_0^C D_t^{\alpha} x(t) \in P_{K_1}(x(t) - \rho M(x(t), y(t)) - \rho a) - x(t) + G_1(x(t)), \quad \text{for } a.e. \, t \in [0, h], \\ x(0) = x_0, \\ {}_0^C D_t^{\alpha} y(t) \in P_{K_2}(y(t) - \lambda N(y(t), x(t)) - \lambda b) - y(t) + G_2(y(t)), \quad \text{for } a.e. \, t \in [0, h], \\ y(0) = y_0, \end{cases} \tag{1.4}$$

where ${}_0^C D_t^{\alpha}$ is the Caputo fractional derivative of order $\alpha \in (0, 1]$, P_{K_1} and P_{K_2} are two projection operators, K_1 and K_2 are two closed convex subsets of two separable Hilbert spaces X and Y respectively, $\rho > 0$ and $\lambda > 0$ are two constants, $x, x_0, a \in X$, $y, y_0, b \in Y$, M: $X \times Y \to X$ and N: $Y \times X \to Y$ are two nonlinear mappings, G_1: $X \to 2^X$ and G_2: $Y \to 2^Y$ are two set-valued

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