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Two population three-player prisoner's dilemma game

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ABSTRACT

Due to the computational advantage in symmetric games, most researches have focused on the symmetric games instead of the asymmetric ones which need more computations. In this paper, we present prisoner's dilemma game involving three players, and suppose that two players among them agree against the third player by choosing either to cooperate together or to defect together at each round. According to that assumption, the game is transformed from the symmetric three- player model to asymmetric two-player model such that, the identities of the players cannot be interchanged without interchanging the payoff of the strategies. Each strategy in the resulting model is expressed with two state automata. We determine the payoff matrix corresponding to the all possible strategies. We noticed that, for some strategies, it is better to be a player of the first type (independent player) than being of the second type (allies).

1. Introduction

Game theory has become a key tool across many disciplines. The prisoner's dilemma (PD) is a traditional game model for the study of decision-making and self interest [1,2]. It is only one of many illustrative examples of the logical reasoning and complex decisions involved in game theory. The mechanisms that drive the (PD) are the same as those that are faced by marketers, military strategists, poker players, and many other types of competitors [3–5] This dilemma can multiply into hundreds of other more complex dilemmas. A plethora of disciplines have studied the game, including artificial intelligence, economics [6,7], biology [8], physics, networks [9], business [10], mathematics[11,12], philosophy, public health, ecology, traffic engineering [13], sociology and computer science [14,15].

In the prisoner's dilemma, two players are faced with a choice, they can either cooperate or defect. Each player is awarded points (called payoff) depending on the choice they made compared to the choice of the opponent. Each player's decision must be made without knowledge of the other player's next move. There can be no prior agreement between the players concerning the game. If both players cooperate they both receive a reward, *R*. If both players defect they both receive a punishment, *P*. If one player defects and the other cooperate, the defector receives a reward, *T* the temptation to defect,

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while the player who cooperated is punished with the sucker's payoff, S [16]. We can represent the payoff matrix as the following:

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \tag{1}$$

where, $T > R > P > S$ should be satisfied [17].

If a rational player thinks that his/her opponent will cooperate, then he will defect to receive a reward, T points as opposed to the cooperation which would have earned him/her only, R points. Moreover if the rational player thinks that his/her opponent will defect, he will also defect and receive, P points rather than cooperate and receive the sucker's payoff of, S points. Therefore, the rational decision is to always defect [18]. But assuming the other player is also rational he/she will come to the same conclusion as the first player. Thus, both players will always defect, earning rewards of, P points rather than , R points that mutual cooperation could have yielded. Therefore, defection is the dominant strategy for this game (the Nash Equilibrium). This holds true as long as the payoffs follow the relationship $T > R > P > S$, and the gain from mutual cooperation is greater than the average score for defection and cooperation, $R > \frac{S+T}{2}$.

The iterated prisoner's dilemma (*IPD*) is an interesting variant of (*PD*) where, the dominant mutual defection strategy relies on the fact that it is a one shot game with no future. The key of the (IPD) is that the two players may meet each other again, and this allows the players to develop their strategies based on the previous game interactions [19]. Therefore, a player's move now may affect how his/her opponent behaves in the future and thus affect the player's future payoffs, and this removes the single dominant strategy of mutual defection because, the players use more complex strategies which depend on the game history to maximize the payoffs that they receive. In fact, under the correct circumstances mutual cooperation can emerge [10,20].

Xia et al. have focused on the weak prisoner's dilemma on random and scale-free (SF) networks, and have shown that degree-uncorrelated activity patterns on scale-free networks significantly impair the evolution of cooperation, and they studied how the heterogeneous coupling strength affects the evolution of cooperation in the prisoner's dilemma game with two types of coupling schemes (symmetric and asymmetric ones) [21]. In addition, the symmetric coupling strength setup leads to the higher cooperation when compared to the asymmetric, that is, the asymmetric coupling loses the evolutionary advantage. Their results convincingly demonstrated that the emergence or persistence of cooperation within many real-world systems can be accounted for by the interdependency between meta-populations or sub-systems [22]. Moreover, they put forward an improved traveler's dilemma game model on two coupled lattices to investigate the effect of coupling effect on the evolution of cooperation based on the traveler's dilemma game, where the coupling effect between two lattices is added into the strategy imitation process, and they indicated that the cooperation behavior can be greatly varied when compared to those obtained on the traditionally single lattices [23]. Their results are surprisingly conducive to understanding the cooperation behavior of traveler's dilemma game within many real world systems, especially for coupled and interdependent networked systems. They integrate the coupling effect between corresponding players on two lattices, and noticed that the coupling or correlation strength between two lattices will observably influence the process of strategy imitation, and further change the persistence and emergence of cooperation in the whole system [24]. Also Perc and Szolonki were interested in studying the enhancement of cooperation, and the impact of diverse activity patterns on the evolution of cooperation in evolutionary social dilemmas [25–28].

Wang et al. studied the evolution of public cooperation on two interdependent networks that are connected by means of a utility function, which determines to what extent payoffs in one network influence the success of players in the other network [9,29]. Also, they have shown that the percolation threshold of an interaction graph constitutes the optimal population density for the evolution of public cooperation, and they have demonstrated this by presenting outcomes of the public goods game on the square lattice with and without an extended imitation range, as well as on the triangular lattice [30–32] importantly, they have found that for cooperation to be optimally promoted, the interdependence should stem only from an intermediate fraction of links connecting the two networks, and that those links should affect the utility of players significantly [33]. Recently, they have studied the evolution of cooperation in the public goods game on interdependent networks that are subject to interconnectedness by means of a network-symmetric definition of utility. Strategy imitation has been allowed only between players residing on the same network, but not between players on different networks. They have shown first that, in general, increasing the relevance of the average payoff of nearest neighbors on the expense of individual payoffs in the evaluation of utility increases the survivability of cooperators [34,35]. They showed that the interdependence between networks self-organizes so as to yield optimal conditions for the evolution of cooperation [36].

Game theory has been extended into evolutionary biology, which has generated great insight into the evolution of strategies under both biological and cultural evolution. The replicator equation, which consists of sets of differential equations describing how the strategies of a population evolve over time under selective pressures, has also been used to study learning in various scenarios [37–39]. There are various approaches to construct dynamics in repeated games [40–42]. Kleimenov and Schneider proposed approach of constructing dynamics in the repeated three-person game to give a tool for solving various optimization problems, for example, the problem of minimizing time of using abnormal behavior types. In their approach, two players act in the class of mixed strategies and the third player acts in the class of pure strategies [43,44]. Matsushima and Ikegami discussed the similarity between a noisy $2p - IPD$ and a noiseless $3p - IPD$ game where the role of noise in