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Consensus of multi-agent system with distributed control on time scales

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ABSTRACT

In this paper, we investigate consensus problem of multi-agent system with distributed control on time scales, which simultaneously includes discrete time delays. Due to Lyapunov stability method and theory of calculus on time scales, sufficient conditions are derived for reaching the globally exponential consensus of the considered system. Finally, simulation examples are given to illustrate the effectiveness of our theoretical results.

1. Introduction

In recent years, consensus/synchronization problems for networks of dynamic multi-agent systems have gained much attention due to its broad applications in many fields and abundant scientific researches have been achieved such as flocking [1–3], formation control [4,5], distributed sensor networks [6] etc. The basic goal of consensus problem is to reach an agreement of all agents under certain controls such as impulsive control, adaptive control, distributive control [7–9], etc.

Consensus control is one of the most critical problem of coordination control whose goal is to ensure that every agent in this system reach an agreement on velocity or state in a finite time. In past decades, much study have been focused on this problem in continuous-time. In [10], Lei et al. proposed a network-based consensus control protocol and each agent will be remotely controlled by employing a communication network. The pairwise output synchronization problem of heterogeneous linear multi-agent systems is proposed and the state synchronization is achieved in jointly connected graph in [11]. Yu et al. considered the consensus problem of fractional-order multi-agent systems and present the detailed analysis in [12]. Consensus problems in discrete-time have also been largely investigated in past a few decades. Synchronization of discrete-time multi-agent systems are studied by using Riccati design in [13]. Xu et al. solved the discrete-time leader-following consensus problem by proposing the consensus protocol based on distributed observer in [14]. A class of consensus protocols are proposed whose goal is to make each agent reach an agreement in [15].

As mentioned above, most of the literatures dealt with the consensus problem in continuous-time and discrete-time, respectively. But there exist abundant of network systems that contains both continuous-time and discrete-time simultaneously. For instance, some neurons in brain are active during the day, and they will be the status of stagnancy when they slept, while they will be activated anew in next day, cycling like that. Thus, it is necessary and significant to consider these two cases together in network systems.

Theory of time scale was introduced by S. Hilger in 1988 which aimed at unifying the continuous and discrete analysis [22]. Every rd-continuous function on time scale $\mathbb{T}$ shall be a piece-wise continuous function on $\mathbb{R}$ from the perspective

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of calculus on time scales in actual. It has tremendous potential for theoretical analysis and applications studied in neural networks [16,17], population dynamics, economics and so on. In [18], Zhou et al. studied the exponential stability of the equilibrium point for the considered neural network by employing the calculus on time scales. Cheng and Cao [19] investigated the global exponential synchronization problem with discrete-time delays based on the calculus on time scales and derived several sufficient conditions to reach the synchronization. Shen and Cao derived a consensus criteria for a class of multi-agent system on time scales in [20]. In view of the disadvantage that discrete time delays (or more generally say, discrete time-varying delays) can only make use of its historical information at one unique moment, in this paper, we adopt the integral distributed communication time delay besides discrete time delay which could make full use of the historical delay information in the time interval $[t-\tau, t]$, thus it contributes to portraying and quantifying the intricate influence arising in nonlinear dynamic systems.

Motivated by the consensus problem proposed in [19,21], we consider

$$x_i^{\Delta}(t) = f(x_i(t), x_i(t-\tau)) + u_i(t), \quad i = 1, 2, \ldots, N, \tag{1.1}$$

with

$$u_i(t) = \sum_{v_j \in N_i} a_{ij} \left[\int_{t-\tau}^{t} K(t-s) x_j(s) \Delta s - \int_{t-\tau}^{t} K(t-s) x_i(s) \Delta s \right], \quad i = 1, 2, \ldots, N. \tag{1.2}$$

By Lyapunov stability method and theory of calculus on time scales, we obtain the consensus of multi-agent system with discrete and finite-distributed delays on time scales.

The rest of this paper is organized as follows: in Section 2, some preliminaries and lemmas are presented. Distributed delay consensus of multi-agents system on time scales are discussed in Section 3. Simulation results are given in Section 4. Conclusion is stated in Section 5.

2. Preliminaries

In this section, some basic definitions and lemmas are presented which will be used to the following sections.

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers. Assume that $0 \in \mathbb{T}$ and unbounded above, i.e., $\sup \mathbb{T} = +\infty$. The forward and backward jump operator $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ are defined by $\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}$ and $\rho(t) := \sup\{s \in \mathbb{T} : s < t\}$, respectively. We put $\inf \emptyset = \sup \mathbb{T}$ (i.e., $\sigma(t) = t$ if $\mathbb{T}$ has a maximum t) and $\sup \emptyset = \inf \mathbb{T}$ (i.e., $\rho(t) = t$ if $\mathbb{T}$ has a minimum t), where $\emptyset$ denotes the empty set. $[a, b]_{\mathbb{T}} := [a, b] \cap \mathbb{T}$.

t is called right-scattered if $\sigma(t) > t$ while t is called left-scattered if $\rho(t) < t$. t is called right-dense if $\sigma(t) = t$ while t is called left-dense if $\rho(t) = t$. If $\mathbb{T}$ has a left-scattered maximum m, then we define $\mathbb{T}^k = \mathbb{T} - m$, otherwise, $\mathbb{T}^k = \mathbb{T}$. The graininess function $\mu(t) : \mathbb{T}^k \to [0, +\infty)$ is defined by $\mu(t) := \sigma(t) - t$. If $f : \mathbb{T} \to \mathbb{R}$ is a function, then the function $f^{\sigma} : \mathbb{T} \to \mathbb{R}$ is defined by $f^{\sigma}(t) = f(\sigma(t))$ for all $t \in \mathbb{T}^k$, i.e., $f^{\sigma} = f \circ \sigma$.

A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous provided it is continuous at right-dense point of $\mathbb{T}$ and the left side limit exists (finite) at left-dense point of $\mathbb{T}$. The set of all rd-continuous functions on $\mathbb{T}$ is denoted by $C_{rd} = C_{rd}(\mathbb{T}, \mathbb{R})$.

A function $p : \mathbb{T} \to \mathbb{R}$ is regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$ holds. The set of all regressive and rd-continuous functions on $\mathbb{T}$ is denoted by $\mathcal{R} = \mathcal{R}(\mathbb{T}, \mathbb{R})$.

A matrix $\aleph > 0$ means that $\aleph$ is symmetric and positive definite. A matrix $\Re < 0$ means that $\Re$ is symmetric and negative definite. The Kronecker product of matrices $\aleph_1 \in \mathbb{R}^{m \times n}$ and $\aleph_2 \in \mathbb{R}^{p \times q}$ is the matrix that each entry of $\aleph_1$ times $\aleph_2$ in $\mathbb{R}^{mp \times nq}$ and denoted by $\aleph_1 \otimes \aleph_2$.

Definition 1 (Bohner and Peterson [22]). Assume $f : \mathbb{T} \to \mathbb{R}$ is a function and let $t \in \mathbb{T}^k$. Then $f^{\Delta}(t)$ is the number, with the property that given any $\varepsilon > 0$, there is a neighborhood U of t (i.e., $U = (t-\delta, t+\delta)_{\mathbb{T}}$ for some $\delta > 0$) such that

$$|[f^{\sigma}(t) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]| \le |\sigma(t) - s| \quad \text{for all} \quad s \in U,$$

then $f^{\Delta}(t)$ is called the delta (or Hilger) derivative of f at t. We say that f is delta differentiable (or in short: differentiable) at t provided $f^{\Delta}(t)$ exists. For all $t \in \mathbb{T}^k$, we can get $f^{\sigma}(t) = f(t) + \mu(t) f^{\Delta}(t)$.

Remark 1. There are two special cases:

(i) If $\mathbb{T} = \mathbb{R}$, then $f : \mathbb{R} \to \mathbb{R}$ is delta differential at $t \in \mathbb{R}$ iff

$$f'(t) = \lim_{s \to t} \frac{f(t) - f(s)}{t - s} \quad \text{exists},$$

i.e., iff f is differentiable (in the ordinary sense) at t. In this case, we have $f^{\Delta}(t) = f'(t)$.

(ii) If $\mathbb{T} = \mathbb{Z}$, then $f : \mathbb{Z} \to \mathbb{R}$ is delta differential at $t \in \mathbb{Z}$ with

$$f^{\Delta}(t) = \frac{f^{\sigma}(t) - f(t)}{\mu(t)} = f(t+1) - f(t) = \triangle f(t),$$

where $\triangle$ is the usual forward difference operator defined by the last equation above.

Lemma 1 (Bohner and Peterson [22]). *Assume $f, g : \mathbb{T}^k \to \mathbb{R}$ are differentiable at $t \in \mathbb{T}^k$. Then*