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Pricing variance swaps under stochastic volatility and stochastic interest rate

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A B S T R A C T

In this paper, we investigate the effects of imposing stochastic interest rate driven by the Cox–Ingersoll–Ross process along with the Heston stochastic volatility model for pricing variance swaps with discrete sampling times. A dimension reduction mechanism based on the framework of Little and Pant (2001) is applied which later reduces to solving two three-dimensional partial differential equations. A semi-closed form solution to the fair delivery price of a variance swap is obtained via the derivation of characteristic functions. Practical implementation of this hybrid model is demonstrated through numerical simulations.

1. Introduction

Volatility is a measure of the price fluctuation of a financial instrument over time. However, volatility/variance has become a class of trading assets in its own right in the past twenty years. In late 1990s, Wall Street firms started trading variance swaps, forward contracts written on the realized variance. Basically, variance swaps are categorized under volatility derivatives which are financial derivatives and their values depend on the future levels of volatility. According to Demeterfi et al. [8], volatility derivatives are traded for decision-making between long or short positions, trading spreads between realized and implied volatility, and hedging against volatility risks. The utmost advantage of volatility derivatives is their capability in providing direct exposure towards the assets volatility without being burdened with the hassles of continuous delta-hedging. The measures of volatility involved can be categorized into three main areas which are historical volatility, implied volatility and model-based volatility. Historical volatility is mainly related to previous standard deviation of financial returns involving a specified time period. An example of a volatility derivative written on this historical volatility measure is the futures on realized variance. The implied-volatility ascertain the volatility by matching volatilities from the market and some specific pricing model. The VIX estimate this type of volatility measure of the S&P 500 index. Finally, the model-based volatility are defined in the class of stochastic volatility models such as [17,27] and others.

Researchers working in the field concerning volatility derivatives have been focusing on developing suitable methods for estimating values of variance swaps. In addition, incorporation of stochastic volatility into the models of pricing and

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hedging variance swaps have also been a trend in the recent literature. For example, based on the Heston stochastic volatility model, Grunbichler and Longstaff [14] developed a pricing model for options on variance. One important finding was the contrast characteristics between volatility derivatives and usual equity options on traded assets. In 1998, Carr and Madan [5] combined static replication using options with dynamic trading in the futures to price and hedge certain volatility contracts without specifying the volatility process. The principle assumptions were continuous trading and continuous semi-martingale price processes for the future prices. Selection of a payoff function which diminished the path dependence property ensured that the investor's joint perception regarding volatility and price was also taken into consideration. Further, Demeterfi et al. [8] proved that a variance swap could be reproduced via a portfolio of standard options. The requirements specified were continuity of exercise prices for the options and continuous sampling times for the variance swaps. However, it was later noted by Heston and Nandi [18] that specifying the mean reverting square root process has the disadvantage of unobservable underlyings. Thus, the latter proposed a user friendly model by working on the discrete-time GARCH volatility process with parametric specifications. This model had the advantage of real market practicability, as well as the capability to hedge various volatility derivatives using only a single asset. In 2007, Elliott et al. [12] constructed a continuous-time Markovian-regulated version of the Heston stochastic volatility model to distinguish the states of a business cycle. Analytical formulas were obtained using the regime-switching Esscher transform and price comparisons were made between models with and without switching regimes. Their results showed that the prices of variance swaps implied by the regime-switching Heston stochastic volatility model were significantly higher than those without switching regimes. One important characteristic shared among these researchers was the assumption of continuous sampling time which is actually an inclination with the discrete sampling reality in financial markets. In fact, options of discretely variance swaps were mis-valued when the continuous sampling were used as approximations, because these continuous approximations produce non-negligible inaccuracies in certain sampling periods as discussed by Bernard and Cui [1], Elliott and Lian [10], Little and Pant [22], and Zhu and Lian [30]. It is worth mentioning that stochastic models have not only been used in quantitative finance in terms of pricing financial derivatives, but also gained popularity in other field, see, for example, [23,24,28].

In addition to previously mentioned analytic approaches, Little and Pant [22] explored the finite difference method in the numerical approaches via dimension-reduction approach and contributed to high efficiency and accuracy for discretely sampled variance swaps. In addition, Windcliff et al. [29] investigated the effects of employing the partial integro differential equation on constant volatility, local volatility and jump diffusion-based volatility products. Their delta-gamma hedging models accomplishes less liability and is effective for discontinuous market traits and ordinary counter-party writing instruments. The work of Little and Pant was extended by Zhu and Lian [30] through incorporating Heston two-factor stochastic volatility for pricing discretely sampled variance swaps. Levels of validity for short periods when using the continuous-time sampling were provided through significant errors, along with analytical hedging derivations and numerical simulations. A recent study was conducted by Bernard and Cui [1] on analytical and asymptotic results for discrete sampling variance swaps with three different stochastic volatility models.

Despite the popularity of pricing discrete variance swaps in the literature reviewed above, an approach for determining the price of discrete variance swaps based on stochastic volatility and stochastic interest rate has not yet seen. The assumption of constant interest rates in the existing literatures in pricing variance swaps were unrealistic in modeling the real market phenomena. Thus, the novelty of our work in the current paper is to price discrete variance swaps by incorporating stochastic interest rate along with stochastic volatility. In this way, we can not only reduce the inaccuracies resulting from continuous sampling time for pricing variance swaps, but also provide a better market characterization with stochastic interest rate.

In the past three decades, many authors have considered modeling stochastic interest rate and its applications in pricing financial derivatives using stochastic approaches. Generally, the modeling trend can be seen as developing from unobservable rates such as spot rates to market rates regularly practiced by financial institutions. Elliott and Siu [11] pointed out that the stochastic interest rate models should be capable of providing a practical realization of the fluctuation property, as well as adequate tractability. They derived exponential affine form of bond prices with elements of continuous-time Markov chain using enlarged filtration and semi-martingale decompositions. In addition, Grzelak and Oosterlee [15] examined correlation issues of European products pricing with the Heston–Hull–White and Heston–CIR hybrid models. Recently, Kim et al. [20] showed that incorporation of stochastic interest rates into a stochastic volatility model gave better results compared with the constant interest rate case in any maturity. They proposed a model which was a combination of the multi-scale stochastic volatility model from [13] and the Hull–White interest rate model. The call option price approximation for this mixed model was obtained via derivation of the leading order and the first order correction prices using Fouque's multiscale expansion method and operator specifications for the correction terms. In a quite recent paper [26], Shen and Siu investigated the effects of stochastic interest rates and stochastic regime-switching volatility for the pricing of variance swaps. However, only continuous sampling approximation in an integral form were formulated for variance swap rates in these research.

In this paper, a hybridization of the Heston stochastic volatility model and the CIR stochastic interest rate model is employed to investigate its effects on the pricing rates of variance swap with discrete sampling. This hybrid model extends the work of Zhu and Lian [30] where stochastic interest rates were ignored. A semi-closed form solution to the fair delivery price of a discretely sampled variance swap is obtained via the dimension reduction technique and derivation of characteristic functions. Comparison between the results obtained from our numerical analysis with those results from other existing models using constant interest rates indicates that our model fills the gap left in the literature on pricing variance swaps. As