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A numerical investigation of incompressible viscous flow in a helical square annulus

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A B S T R A C T

In this article incompressible viscous flow in a helical square annulus is investigated numerically. The governing equations including the continuity and Navier–Stokes equations are written in an orthogonal helical coordinate system. This provides a uniform computational domain with a capability of using an exact second order finite difference method based on the projection algorithm to discretize the governing equations. The numerical results obtained study the effects of different non-dimensional parameters such as torsion, curvature, Reynolds number and aspect ratio on the secondary flow, axial velocity and friction factor in detail.

1. Introduction

Non-straight pipes are used extensively in various engineering systems such as piping systems, steam generators, heat exchangers, air conditioners and so on. Centrifugal effects due to the curvature in curved ducts make the physical aspect of the flow much more complicated. Furthermore, in a helical duct the presence of torsion causes asymmetry in the flow pattern comparing with the symmetric flow pattern taking place in curved pipes with zero torsion. Secondary flow is one of the most important aspects of the flow in curved pipe systems, and understanding its characteristics is useful in the designing of corresponding systems.

After the pioneer study of Dean [1], many researchers have developed the work on the flow and heat transfer in curved and helical pipes. Among the numerical studies, Chen and Jan [2] investigated the effect of curvature, torsion and the inclined angle of cross section in helical square ducts and noticed that the torsion has more pronounced effect on the secondary flow rather than the axial one. In addition, they discovered that Deans instability due to the secondary flow can be avoided due to the effect of torsion and/or inclined angle. A numerical study conducted by Chen and Zhang [3] has examined the combined effects of rotation (Coriolis force), curvature (centrifugal force) and heating/cooling (centrifugal-type buoyancy force) on the fluid flow and mixed convection heat transfer in a rotating curved pipe. Liberto and Ciofalo [4] analyzed turbulent heat transfer in curved pipes numerically and showed that the time averaged results exhibit Dean circulation and a strong velocity and temperature stratification in the radial direction. The numerical study by Chen et al. [5] has taken into account both the nature of flow behaviors and the characteristics of heat transfer in a rotating helical square duct. They investigated the flow structure and temperature distribution with the ratio of the Coriolis to the centrifugal force and torsion in detail. Amani and Nobari [6] numerically simulated developing incompressible viscous flow and heat transfer

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Nomenclature

A, B	auxiliary parameter
De	Dean number, $De = \sqrt{\kappa}Re$
D_H	hydraulic diameter, $D_H = D_o - D_i$
D_o	outer duct diameter
D_i	inner duct diameter
f	friction factor
Fr	friction factor ratio
L	length of computational domain
$\vec{n}$	unit normal vector
P	pressure
P_H	helical pitch
$\vec{Q}$	position vector
$(r\,q\,s)$	orthogonal coordinate system
$\vec{R}$	the vector describes the centerline of a helical duct
Re	Reynolds number
R_H	radius of curvature
t	time
$\vec{T}$, $\vec{N}$, $\vec{B}$	tangent, normal and binormal vectors respectively
U	mean axial velocity
v_s, v_q, v_r	velocity components in s, q, r directions, respectively
v_s, v_x, v_y	velocity components in s, x, y directions, respectively
$(x\,y\,s)$	auxiliary non-orthogonal coordinate system
v	velocity vector

Greek symbols

δ_r	aspect ratio D_i/D_o
κ	curvature
ν	kinematic viscosity
ρ	density
τ	torsion
ϕ	angle of rotation
λ	auxiliary parameter
Φ	number of turns of helix

Subscripts

Γ	boundary

Superscripts

$*$	artificial
$\sim$	dimensional

in curved pipes to analyze the entropy generation for the thermodynamic optimization in the entrance region at a constant wall temperature. Nobari et al. [7] studied developing incompressible viscous fluid flow and heat transfer in a curved annular pipe and indicated that the friction factor and the Nusselt number in a curved annular pipe are both proportional to the square root of Dean number. Later on Nobari and Malvandi [8] numerically studied fully developed fluid flow in a helical annulus pipe and investigated the effects of different physical parameters such as aspect ratio, torsion, curvature and Reynolds number on the flow field. Their results indicated that a decrease in the aspect ratio and torsion lead to the increase of friction factor at a given Dean number. Nobari and Rajaei [9] presented a numerical study of developing incompressible flow in concentric and eccentric curved square annuli. They indicated that the friction factor in an eccentric curved square annulus decreases with the eccentricity. The study by Wang [10] took into account the effects of torsion on the flow field in helical pipes for the first time. He used the perturbation method based on a non-orthogonal helical coordinate system to solve helical flow problems with the small curvature and torsion analytically. His results indicate a first-order effect of torsion on the flow field. However, Germano [11,12] performed the same study by introducing an orthogonal coordinate system and showed that the effect of torsion on a helical pipe flow is the second-order while the effect of curvature is the first-order. The study of Chen et al. [13] on the flow in rotating helical pipe with elliptical cross section led to the result that shows the number of secondary flow cells and their intensity depends on the ratio of Coriolis force to centrifugal force.

There are many experimental studies about the curved and helical pipes. Among them, Mori et al. [14] have used analytical and experimental methods for forced convective heat transfer in a curved channel with a square cross section. They indicated the intensity of secondary flow depend upon the value of the Dean number. Williams et al. [15] experimentally