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Option pricing in jump diffusion models with quadratic spline collocation

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ABSTRACT

In this paper, we develop a robust numerical method in pricing options, when the underlying asset follows a jump diffusion model. We demonstrate that, with the quadratic spline collocation method, the integral approximation in the pricing PIDE is intuitively simple, and comes down to the evaluation of the probabilistic moments of the jump density. When combined with a Picard iteration scheme, the pricing problem can be solved efficiently. We present the method and the numerical results from pricing European and American options with Merton's and Kou's models.

1. Introduction

It is well-known that the simple Black–Scholes model cannot explain the implied volatility smile seen in the calibration of the volatility parameter to market prices. Large jumps in asset returns have also been observed, and these discontinuities are arguably not reflected in a pure diffusion model.

Several models have been developed as alternatives to or extensions of the Black–Scholes model. The local volatility approach, proposed in [11], generalizes the Black–Scholes model by considering a deterministic time-dependent and price-dependent volatility function. This model is theoretically attractive, as the local volatility function can be computed when a smooth set of option prices in all strikes and maturities is known. In reality, a numerical optimization method (see for example [8]) is required, as there is only a restricted set of options that trade enough, so that the quoted prices are accurate enough for these methods to work. As was pointed out in [2], the local volatility approach alone does not produce stationary implied volatilities, in contrast with market experience. Stochastic volatility models, such as those in [14] or [15], have been proposed. As multi-factor models, these models are numerically more expensive.

Another alternative to the classical Black–Scholes model is a jump diffusion model, due to [18]. The idea is to model asset returns by the usual Wiener process combined with a compound Poisson process, the latter corresponding to the "jumps". By varying the jump parameters, one can obtain volatility shapes, and control the skewness and kurtosis of the log asset return. A number of studies (for example, [3,4]) suggest the addition of jumps to stochastic volatility models.

When the asset follows a jump diffusion model where the jump density is log-normally distributed [18], closed-form formulas can be obtained for European options' prices. For other jump size distributions, closed-form solutions may not be guaranteed. Pricing exotic options in general requires numerical methods as well.

The option prices in a jump model satisfy a partial integro-differential equation (PIDE). The integral is performed over the entire grid, and this non-locality poses numerical challenges. Different numerical treatments have been proposed. Explicit

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methods or explicit treatment of the integral, such as those proposed in [1] or [19], suffer from severe stability and convergence constraints. An FFT-ADI (Fast Fourier Transform - Alternating Direction Implicit) approach is developed in [2], which is unconditionally stable and has second order convergence. But as noted in both [5,10], it is not clear how this method can be extended to pricing American options.

An explicit FFT treatment of the integral term together with a fixed-point iteration is suggested in [10], and has second order convergence. The method, while robust and unconditionally stable, suffers from a common problem in Fourier-based methods. The effect of wrap-around errors resulting from non-periodicity of option values can be large. The remedy for this, suggested in [10], is to enlarge the grid to the point that the wrap-around errors will be non-material. Enlarging the grid can be expensive especially in higher-dimensions. For Kou's model of the jump density, a much larger grid than that of Merton's is necessary to avoid large wrap-around errors. It was also reported in [10] that convergence can be slow for Kou's model, when numerical FFT methods are used.

The Fourier family methods also include the Fourier space timestepping method, developed in [16]. These Fourier methods are effective and robust, yet they usually require special attention to the grids. A uniform grid is usually required for FFTs, sometimes leading to a waste of computational resources in regions that are not important. In addition, a log transform is needed to utilize the FFT methods. That means one has to maintain two grids, and interpolation between the two grids is needed to communicate information. When one does timestepping in the original S-space instead of the Fourier space (the latter suggested in [16]), up to two interpolations can be required in each timestep. Furthermore, it is not easy to generalize the Fourier methods for the pricing of various exotic options.

There are other numerical approaches in the literature. Clever change of variable techniques have been proposed in [5], which computes the integral in Merton's model by solving the heat equation along an artificial dimension, and for Kou's model enables fast valuation of the integral term in linear time. While these are attractive, the transformations are specific to the aforementioned two models (and their related families).

In this paper, we develop a quadratic spline collocation method in the pricing problem under a finite activity jump diffusion model. The evaluation of the integral term is reduced to the computation of the probabilistic moments of the jump density, and works best for jump densities that have analytically tractable partial moments. For a fixed non-adaptive (uniform or non-uniform) grid, our method requires only a pre-computation of certain integral matrices. To the authors' knowledge, no prior work has been done on spline collocation methods in jump diffusion models.

2. PDE/PIDE formulation

2.1. Model

Let $S(t)$ be the price of an asset. In the jump diffusion framework, $S(t)$ is assumed to follow the dynamics

$$\frac{dS}{S} = \mu dt + \sigma dW(t) + d\left(\sum_{i=1}^{N(t)}(J_i - 1)\right), \tag{1}$$

where μ and σ are drift and volatility parameters respectively, W_t is a Wiener process, $N(t)$ is Poisson with rate λ, and J_i is the size of the ith jump, for $i = 1, \dots, N(t)$. We assume that the J_i's are non-negative i.i.d. with some distribution g.

It is clear that some modeling assumptions on g are needed. In this paper, we focus primarily on two models that are commonly considered studied in the literature, namely Merton's and Kou's models, with the density g defined, respectively, by

$$\text{(Merton)} \quad g(x) = \frac{\exp\left(-\dfrac{(\log(x)^2 - \mu)}{2\gamma^2}\right)}{\sqrt{2\pi}\gamma x} \tag{2}$$

where $\gamma > 0, \mu \in \mathcal{R}$, and

$$\text{(Kou)} \quad g(x) = \begin{cases} p\eta_1 \dfrac{\exp(-\eta_1 \log(x))}{x} & \text{for } x \geq 1, \\ (1-p)\eta_2 \dfrac{\exp(\eta_2 \log(x))}{x} & \text{for } x < 1. \end{cases} \tag{3}$$

where $\eta_1 > 1$, $\eta_2 > 0$, and $0 < p < 1$. The specification (2), originally considered in [18], is the first of its kind, and models a jump size density by a lognormal distribution (equivalently, it models $\log(J)$ by a normal distribution). On the other hand, the assumption (3), considered in [17], models $\log(J)$ by a double exponential distribution.

We remark that calibration of the jump parameters is out of the scope of this study.

Let $V(S, \tau)$ be the value of a contingent claim that depends on S and backward time $\tau = T - t$, where T is the expiration time of the contract. Based on the stochastic differential equation (SDE) (1), it can be shown (see [18]) that V satisfies the PIDE

$$\frac{\partial V}{\partial \tau} = \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - \lambda\kappa)S\frac{\partial V}{\partial S} - (r+\lambda)V + \lambda\int_0^\infty V(S\eta)g(\eta)d\eta \tag{4}$$