Applied Mathematics and Computation 279 (2016) 62–75

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A lattice Boltzmann model for the ion- and electron-acoustic solitary waves in beam-plasma system

Huimin Wang*

College of Applied Mathematics, Jilin University of Finance and Economics, Changchun 130117, PR China

ARTICLE INFO

Keywords:
Lattice Boltzmann model
The ion- and electron-acoustic solitary wave
Beam-plasma

ABSTRACT

In this paper, a lattice Boltzmann model for the ion- and electron-acoustic solitary waves in beam-plasma system is proposed. By using the Chapman–Enskog expansion and the multi-scale time expansion, a series of partial differential equations in different time scales are obtained. By selecting the appropriate moments of the equilibrium distribution functions, the macroscopic equations are recovered. In numerical examples, we simulate the propagation of the ion- and electron-acoustic solitary waves. Numerical results show that the lattice Boltzmann method is an effective tool for the study of the ion- and electron-acoustic solitary waves in plasma.

1. Introduction

In several regions of the Earth's magnetosphere, e.g., the bow shock, magnetosheath, magnetopause and on cusp field lines, Polar cap boundary layer, auroral field lines, auroral kilometric radiation source region, and plasma sheet boundary layer, the electrostatic solitary waves have been observed [1]. The frequency range of these electrostatic waves is much higher than the ion-acoustic frequency, but still lower than the Langmuir wave frequency. Therefore, the waves can be considered as a kind of electron-acoustic waves. The studies of electron-acoustic solitary waves in space plasma have practical significance.

The electron-acoustic solitary wave was first discovered in the study of plasma with high ion temperature and low electron temperature. Electron-acoustic frequency is close to the ion plasma frequency, and the phase velocity is close to the electron thermal velocity. The electron-acoustic solitary wave also can generate from the effects of high energy electron beam flow or high temperature electron distribution on the low temperature electron distribution.

Many studies have been spent on the electron-acoustic solitary waves. Some multi-component plasmas models were discussed to explain the solitary waves with negative potentials observed by Viking satellite [2–9]. The studies of Berthomier et al. and El-Taibany showed that solitons with positive polarity could be generated depending upon the beam velocity, temperature and density [10–12]. Verheest et al. showed that even without the electron-beam component, the positive potential electron-acoustic solitons can be generated [13]. The model of Kakad et al. supported the existence of solitons having either positive or negative potentials [14]. Lakhina et al. have a detailed discussion about the earlier models, and further research on the waves in space plasmas [1,15–19]. Even so, the accurate solution of many such problems is still difficult. Therefore the numerical solutions of the problems need to be employed. In this paper, we focus on the simulation of the ion- and electron-acoustic solitary waves in beam-plasma system by using the lattice Boltzmann method (LBM).

* Tel.: +86043184539342.
E-mail address: whm20110605@126.com, whm780921@sina.com

http://dx.doi.org/10.1016/j.amc.2016.01.007

The LBM is a numerical method which originated from a Boolean fluid model named as the lattice gas automata (LGA). Unlike traditional numerical methods which solve macroscopic variables, the LBM is based on the mesoscopic kinetic equation for the particle distribution function [20–24]. By using the ensemble-averaged distribution functions, the Navier–Stokes equations can be found from the lattice Boltzmann equation. Nowadays the LBM has established itself as a powerful tool for the simulation of a wide range of physical phenomena such as the multiphase flows [25–30], multi-component flows [26,31–33], flows of suspensions [34,35], porous media flows [36,37], compressible flows [38–40], the plasma [41], and relativistic hydrodynamics [42]. Additionally, the lattice Boltzmann models have been developed to simulate linear and nonlinear partial differential equations such as nonlinear Schrödinger equation [43–47], Burgers equation [48,49], wave motion equation [50,51], the shallow equation [52], Lorenz equations [53], the Poisson equation [54–56], and the Richards equation [57].

It is assumed that an unmagnetized plasma is affected by a high temperature electron flow. The high temperature electrons velocity meets the Boltzmann distribution. So the low temperature electron fluid satisfies the equations

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}(n_e v_e)=0, \tag{1}$$

$$\alpha_e\left(\frac{\partial v_e}{\partial t}+v_e\cdot\frac{\partial v_e}{\partial x}\right)=\frac{\partial \Phi}{\partial x}-\frac{T_e}{T_h n_e}\frac{\partial n_e}{\partial x}. \tag{2}$$

The low temperature ion fluid satisfies the equations

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_i)=0, \tag{3}$$

$$\left(\frac{\partial v_i}{\partial t}+v_i\cdot\frac{\partial v_i}{\partial x}\right)=-Q_0\frac{\partial \Phi}{\partial x}-\mu_0\frac{T_i}{T_h n_i}\frac{\partial n_i}{\partial x}, \tag{4}$$

where n_e represents electron density, n_i represents ion density, v_e is electron fluid velocity, v_i is ion fluid velocity, Φ is the normalized electrostatic potential, $Q_0=\frac{m_e}{m_i}$ is the mass ratio of electronic and ion, $\alpha_e=\frac{n_{e0}}{n_{h0}}$ represents the ratio of unperturbed cold electron number and hot electron number.

The Poisson equation is

$$\frac{\partial^2 \Phi}{\partial x^2}=n_e+n_h-n_i, \tag{5}$$

where $n_h=n_{h0}\cdot\exp\Phi$. Eqs. (1)–(5) are dimensionless equations. For simplicity, we assume the temperatures of the low temperature electrons and ions are all zero, i.e. $T_e=T_i=0$.

In next section, we propose a lattice Boltzmann model for the ion- and electron-acoustic solitary waves in beam-plasma system; in Section 3, we use the scheme to simulate the propagation of solitary waves; and in Section 4 some conclusions are discussed.

2. Lattice Boltzmann model

2.1. Lattice Boltzmann equation

We define $f_\alpha^\sigma(\mathbf{x},t)$ as the one-particle distribution function with species σ, at position $\mathbf{x}$, time t with velocity $\mathbf{e}_\alpha$. A one-dimensional lattice with unit spacing is used on which each node has b neighbours connected by b links. In this lattice, particles can only reside on the nodes and move to their neighbours along these links in the unit time step. Each node is allowed to host rest particles and possesses b+1 discrete lattice velocities. In this paper, we select $b=2$, therefore, the lattice is a 3-bit lattice.

The lattice Boltzmann equation will be obtained as

$$f_\alpha^\sigma(\mathbf{x}+\mathbf{e}_\alpha,t+1)-f_\alpha^\sigma(\mathbf{x},t)=-\frac{1}{\tau}[f_\alpha^\sigma(\mathbf{x},t)-f_\alpha^{\sigma,eq}(\mathbf{x},t)]+\Omega_\alpha^\sigma(\mathbf{x},t), \tag{6}$$

where $f_\alpha^{\sigma,eq}(\mathbf{x},t)$ is the equilibrium distribution function with species σ, at position $\mathbf{x}$, time t with velocity $\mathbf{e}_\alpha$, τ is the single relaxation time, Ω_α^σ is a non-collision term. We assume the equilibrium distribution $f_\alpha^{\sigma,eq}(\mathbf{x},t)$ meets the conservation conditions

$$\sum_\alpha f_\alpha^{\sigma,eq}(\mathbf{x},t)=\sum_\alpha f_\alpha^\sigma(\mathbf{x},t). \tag{7}$$

The Knudsen number ε is defined as $\varepsilon=\frac{l}{L}$, where l is the mean free path, and L is the characteristic length. It can be taken as the time step Δt [50], thus, the lattice Boltzmann Eq. (6) in physical units is

$$f_\alpha^\sigma(\mathbf{x}+\varepsilon\mathbf{e}_\alpha,t+\varepsilon)-f_\alpha^\sigma(\mathbf{x},t)=-\frac{1}{\tau}[f_\alpha^\sigma(\mathbf{x},t)-f_\alpha^{\sigma,eq}(\mathbf{x},t)]+\Omega_\alpha^\sigma(\mathbf{x},t). \tag{8}$$