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Stability analysis and stabilization for nonlinear continuous-time descriptor semi-Markov jump systems[☆]

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ABSTRACT

This paper investigates the stochastic stability and the state feedback control design for a class of nonlinear continuous-time descriptor semi-Markov jump systems whose transition rates are time-varying, which are more general than the descriptor Markov jump systems. First, by deriving the infinitesimal generator for stochastic Lyapunov functional of descriptor semi-Markov jump systems, a stochastic stability condition is established, which guarantees this kind of systems are regular, impulse-free, have a unique solution, and are stochastically stable. In order to design the state feedback controller, a linear matrix inequality (LMI) stability condition is developed based on the lower and upper bounds of the time-varying transition probability and singular value decomposition approach. Furthermore, the state feedback controller design is developed in terms of LMI approach. Last, numerical examples are given to demonstrate the effectiveness of the obtained methods.

1. Introduction

As a special class of stochastic hybrid systems, Markov jump systems (MJSs) have attracted extensive research attention due to their practical applications in manufacturing, power, aerospace, networked control systems, and so on. Over the last decades, a lot of efforts have been devoted to the analysis and synthesis of MJSs, such as stability and stabilization [1,2], H_∞ control and filtering problems [3–6] for MJSs, the stability analysis [7–9] and finite-time stochastic stabilization [10] of MJSs with partially known transition probabilities, passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time delays [11], the output feedback control of Markovian jump repeated scalar nonlinear systems [12], the sliding mode approaches to uncertain Markovian neutral-type stochastic systems [13] and linear time-delay MJSs with generally incomplete transition rates [14], robust extended dissipative control for sampled-data MJSs [15], etc.

However, MJSs have many limitations in applications, since the sojourn time of a Markov chain is, in general, exponentially distributed, and the results for the MJSs obtained are intrinsically conservative due to constant transition rates. In practice, the transition rates for many systems are not constants, so such systems cannot be modeled by MJSs. Different from the MJSs, semi-Markov jump systems (S-MJSs) are characterized by a fixed matrix of transition probabilities and a matrix of sojourn time probability density functions. Due to their relaxed conditions on the probability distributions, S-MJSs have

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much broader applications than the conventional MJSs, many practical systems, for example, fault-tolerant control systems [16], can be described as S-MJSs. Indeed, most of the modeling, analysis, and design results for MJSs would be special cases of S-MJSs. Thus, the research for S-MJSs is significant in theory and in practice. For example, Hou et al. [17,18] addressed the stability analysis for a class of phase-type S-MJSs, where the sojourn time follows the phase-type distributions. For Weibull probability distributions of sojourn time, Huang and Shi [19,20] discussed the stochastic stability and robust state feedback stabilization for S-MJSs by LMI approach. Wei et al. [21,22] discussed the output feedback control for continuous-time S-MJSs. What's more , with randomly occurring uncertainties and sensor failures, Shen et al. [23] investigated the reliable mixed passive and H_∞ filtering problem for S-MJSs. Zhang et al. [24] gave the concept of semi-Markov kernel, and addressed the stability and stabilization of discrete-time linear S-MJSs via semi-Markov kernel approach.

Descriptor systems, also referred to as singular systems, differential-algebraic systems, generalized state-space systems or semi-state systems, appear in many systems, such as power systems, network analysis, economic systems, biological systems, and so on. In recent years, descriptor systems have been widely studied, such as time domain analysis, admissibility and control synthesis for linear descriptor systems [25,26]. For descriptor systems with the Lipschitz nonlinear function, Lu and Ho [27,28] gave the sufficient LMI conditions that the nonlinear descriptor systems have unique solution and are admissible according to the fixed point principle. The problems of stability and stabilization for time-delay descriptor systems with the Lipschitz nonlinearities were studied in [29,30], however, the existence and uniqueness of the solution to this kind of systems were not considered.

On another research front, descriptor MJSs, as a special class of MJSs, have been widely studied and many good results have been reached [31–41]. Since descriptor MJSs are always existed in power grid and economic systems, the study on descriptor MJSs is important not only in theory but also in practice. For example, the stability analysis and controller design problems for descriptor MJSs and descriptor MJSs with time-delay were discussed in [31–34], respectively. H_∞ filtering for descriptor MJSs with time-delay was discussed in [35–37]. For nonlinear descriptor MJSs, there are also some results. For example, H_∞ state feedback fuzzy control for T-S fuzzy time-delay descriptor MJSs was discussed by Li et al. [38]. With the nonlinearities satisfying Lipschitz condition, Song et al. [39] discussed the stochastic stability and the existence of the unique solution for nonlinear discrete-time descriptor MJSs by LMI approach and implicit function theorem. Without discussing the existence and uniqueness of solution, the stochastic stability and stabilization problems for nonlinear descriptor MJSs with time-delay were discussed respectively by Ding et al. [40] and Long et al. [41].

Although there are many results for nonlinear descriptor systems and descriptor MJSs, and the stability and control design problems for S-MJSs have been received increasing interest, however, the issue of stability analysis and stabilization, the existence of the unique solution for nonlinear continuous-time descriptor S-MJSs have not been studied yet. Since the descriptor S-MJSs are more general than S-MJSs and descriptor MJSs, and the applications for descriptor S-MJSs are more wider than descriptor MJSs, the stability and stabilization problems for nonlinear continuous-time descriptor S-MJSs are important both in theory and practice, which motivates the current research.

In this paper, the stochastic stability and state feedback stabilization problems for nonlinear continuous-time descriptor S-MJSs are investigated. First, in terms of stochastic Lyapunov functional and implicit function theorem, a stability sufficient condition is established, which guarantees the nonlinear continuous-time descriptor S-MJSs are regular, impulse-free, have a unique solution, and are stochastically stable. Depend on the above stability condition, an LMI stability condition is developed based on singular value decomposition of the differential matrix, which can be design a state feedback controller directly. Then, the state feedback stabilization controller is designed. Last, three numerical examples are given to demonstrate the validness of the obtained results.

Notations.Throughout this paper, $X \geq 0$ (or, $X > 0$) means that the symmetric matrix X is semi-positive definite (or, positive definite). I and 0 represent, respectively, the identity matrix and zero matrix with appropriate dimensions. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote, respectively, the n-dimensional Euclidean space, and the set of all $m \times n$ real matrices. The superscript 'T' denotes the transpose of a matrix, diag$\{\cdots\}$ represents a block-diagonal matrix. $\|x\|$ refers to Euclidean norm of the vector x. $\mathbf{E}[\cdot]$ stands for the mathematical expectation. In addition, in symmetric block matrices, $*$ represents as an ellipsis for the terms that are introduced by symmetry, and $sym(X)$ represents $X + X^T$. $(\Omega, \mathcal{F}, \mathcal{P})$ denotes a complete probability space, in which Ω is the sample space, $\mathcal{F}$ is the σ algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$.

2. Preliminaries

Consider the following nonlinear continuous-time descriptor semi-Markov jump system, which is defined in a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$E\dot{x} = A(r_t)x + B(r_t)u + f_{r_t}(t, x), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^p$ is the control input, and r_t, $t \geq 0$ is a continuous-time semi-Markov process taking values in a finite space $\mathcal{N} = \{1, 2, \dots, N\}$ with the following probability transitions:

$$\Pr\{r_{t+h} = j | r_t = i\} = \begin{cases} \lambda_{ij}(h) + o(h), & r_t \text{ jumps from mode } i \text{ to mode } j, \\ 1 + \lambda_{ij}(h)h + o(h), & r_t \text{ stays at mode } i, \end{cases}$$

where $\lambda_{ij}(h) \geq 0$ is transition rate from mode i to mode j for $i \neq j$, and $\lambda_{ii}(h) = -\sum_{j=1, j\neq i}^{N} \lambda_{ij}(h)$, $h > 0$, $\lim_{h\to 0} o(h)/h = 0$. In practice, the transition rate $\lambda_{ij}(h)$ is generally bounded by $\underline{\lambda}_{ij}$ and $\overline{\lambda}_{ij}(\underline{\lambda}_{ij} \leq \overline{\lambda}_{ij})$. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular