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Lacunary ideal convergence of multiple sequences in probabilistic normed spaces

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ABSTRACT

An ideal $\mathcal{I}$ is a family of subsets of positive integers $\mathbb{N}\times\mathbb{N}$ which is closed under finite unions and subsets of its elements. The aim of this paper is to study the notion of lacunary $\mathcal{I}$-convergence of double sequences in probabilistic normed spaces as a variant of the notion of ideal convergence. Also lacunary $\mathcal{I}$-limit points and lacunary $\mathcal{I}$-cluster points have been defined and the relation between them has been established. Furthermore, lacunary-Cauchy and lacunary $\mathcal{I}$-Cauchy, lacunary $\mathcal{I}^*$-Cauchy, lacunary $\mathcal{I}^*$-convergent double sequences are introduced and studied in probabilistic normed spaces. Finally, we provided example which shows that our method of convergence in probabilistic normed space is more general.

1. Introduction

Menger [40] proposed the probabilistic concept of the distance by replacing the number $d(p, q)$ as the distance between points p, q by a probability distribution function $F_{p,\,q}(x)$. He interpreted $F_{p,\,q}(x)$ as the probability that the distance between p and q is less than x. This led to the development of the area now called probabilistic metric spaces. Šerstnev [60] who first used this idea of Menger to introduce the concept of a PN space. In 1993, Alsina et al. [2] presented a new definition of probabilistic normed space which includes the definition of Šerstnev as a special case. For an extensive view on this subject, we refer [1,3,13,21,22,29,34,36,44,49,58,59].

Steinhaus [61] and Fast [16] independently introduced the notion of statistical convergence for sequences of real numbers. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory and number theory. Later on it was further investigated from various points of view. For example, statistical convergence has been investigated in summability theory by (Connor [11], Fridy [18], Šalát [55]), number theory and mathematical analysis by (Buck [4], Mitrinović et al., [42]), topological groups (Çakalli [5,6]), topological spaces (Di Maio and Kočinac [39]), function spaces (Caserta and Kočinac [8]), locally convex spaces (Maddox [38]), measure theory (Cheng et al., [9], Connor and Swardson [12], Miller [41]). Fridy and Orhan [19] introduced the concept of lacunary statistical convergence. Some work on lacunary statistical convergence can be found in [20,25,37,50,52].

Kostyrko et al., [31] introduced the notion of I-convergence as a generalization of statistical convergence which is based on the structure of an admissible ideal I of subset of natural numbers $\mathbb{N}$. Kostyrko et al., [32] gave some of basic properties of I-convergence and dealt with extremal I-limit points. Tripathy and Tripathy [62] introduced the concept of I-convergence of double sequences of real numbers and studied some basic properties of this notion. Mursaleen and Mohiuddine [47]

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and Rahmat [54] studied the ideal convergence in probabilistic normed spaces and Kumar and Kumar [33] studied I-Cauchy and I^*-Cauchy sequences in probabilistic normed spaces. Kumar and Guillén [34] introduced ideal convergence of double sequences in probabilistic normed spaces and proved some interesting results. The notion of lacunary ideal convergence of real sequences was introduced in [10,63] and Hazarika [23,24], introduced the lacunary ideal convergent sequences of fuzzy real numbers and studied some properties. Debnath [14] introduced the notion lacunary ideal convergence in intuitionistic fuzzy normed linear spaces. Yamanci and Gürdal [65] introduced the notion lacunary ideal convergence in random n-normed space. Recently, in [28] Hazarika introduced the lacunary ideal convergent double sequences of fuzzy real numbers and studied some interesting properties of this notion. Further details on ideal convergence we refer to [7,15,26,27,35,43,45,48,51,56,64], and many others.

By a lacunary sequence $\theta = (k_r)$, where $k_0 = 0$, we shall mean an increasing sequence of non-negative integers with $k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $J_r = (k_{r-1}, k_r]$ and we let $h_r := k_r - k_{r-1}$. The space of lacunary strongly convergent sequences $\mathcal{N}_\theta$ was defined by Freedman et al., [17] as follows.

$$\mathcal{N}_\theta = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum\nolimits_{k \in J_r} |x_k - L| = 0, \text{ for some } L \right\}.$$

By the convergence of a double sequence we mean the convergence in the Pringsheim's sense [53]. A double sequence $x = (x_{k,l})$ has a *Pringsheim limit* L (denoted by $P - \lim x = L$) provided that given an $\varepsilon > 0$ there exists an $n \in \mathbb{N}$ such that $|x_{k,l} - L| < \varepsilon$ whenever $k, l > n$. We shall describe such an $x = (x_{k,l})$ more briefly as "*P – convergent*".

In [46] Mursaleen and Edely introduced the two dimensional analogue of natural (or asymptotic) density. Let $K \subset \mathbb{N} \times \mathbb{N}$ and $K(m, n)$ denotes the number of (i, j) in K such that $i \leq m$ and $j \leq n$. Then the lower natural density of K is defined by $\underline{\delta}_2(K) = \liminf_{m,n\to\infty} \frac{K(m,n)}{mn}$. In case, the sequence $(\frac{K(m,n)}{mn})$ has a limit in Pringsheim's sense, then we say that K has a *double natural density* and is defined by $P - \lim_{m,n\to\infty} \frac{K(m,n)}{mn} = \delta_2(K)$.

The double sequence $\overline{\theta} = \theta_{r,s} = \{(k_r, l_s)\}$ is called *double lacunary sequence* if there exist two increasing of integers such that (see [57])

$$k_o = 0, \;\; h_r = k_r - k_{r-1} \to \infty \quad \text{as } r \to \infty$$

and

$$l_o = 0, \;\; \overline{h_s} = l_s - l_{s-1} \to \infty \quad \text{as } s \to \infty.$$

We denote $k_{r,s} = k_r l_s$, $h_{r,s} = h_r \overline{h_s}$ and $\theta_{r,\,s}$ is determined by

$$J_{r,s} = \{(k, l) : \;\; k_{r-1} < k \leq k_r \quad \text{and} \quad l_{s-1} < l \leq l_s\},$$

$$q_r = \frac{k_r}{k_{r-1}}, \quad \overline{q_s} = \frac{l_s}{l_{s-1}} \quad \text{and} \quad q_{r,s} = q_r \overline{q_s}.$$

In this paper we study the concept of lacunary $\mathcal{I}$-convergence of double sequences in probabilistic normed spaces. We also define lacunary $\mathcal{I}$-limit points and lacunary $\mathcal{I}$-cluster points in probabilistic normed space and prove some interesting results.

2. Basic definitions and notations

Now we recall some notations and basic definitions that we are going to use in this paper.

The notion of statistical convergence depends on the density (asymptotic or natural) of subsets of $\mathbb{N}$.

Definition 2.1. A subset E of $\mathbb{N}$ is said to have natural density $\delta(E)$ if

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n} |\{k \leq n : k \in E\}| \quad \text{exists.}$$

Definition 2.2. A sequence $x = (x_k)$ is said to be *statistically convergent* to ℓ if for every $\varepsilon > 0$

$$\delta(\{k \in \mathbb{N} : |x_k - \ell| \geq \varepsilon\}) = 0.$$

In this case, we write $S - \lim x = \ell$ or $x_k \to \ell(S)$ and S denotes the set of all statistically convergent sequences.

Definition 2.3. A family of subsets of $\mathbb{N}$, positive integers, i.e. $I \subset 2^{\mathbb{N}}$ is an ideal on $\mathbb{N}$ if and only if

(i) $\phi \in I$,
(ii) $A \cup B \in I$ for each $A, B \in I$,
(iii) each subset of an element of I is an element of I.

Definition 2.4. A non-empty family of sets $F \subset 2^{\mathbb{N}}$ is a filter on $\mathbb{N}$ if and only if

(a) $\phi \notin F$
(b) $A \cap B \in F$ for each $A, B \in F$,
(c) any superset of an element of F is in F.