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An analytical and numerical study of long wave run-up in U-shaped and V-shaped bays

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ABSTRACT

By assuming the flow is uniform along the narrow long bays, the 2-D nonlinear shallow-water equations are reduced to a linear semi-axis variable-coefficient 1-D wave equation via the generalized Carrier–Greenspan transformation. The run-up of long waves in constantly sloping U-shaped and V-shaped bays is studied both analytically and numerically within the framework of the 1-D nonlinear shallow-water theory. An analytic solution, in the form of a double integral, to the resulting linear wave equation is obtained by utilizing the Hankel transform, and consequently the solution to the tsunami run-up problem is developed by applying the inverse generalized Carrier–Greenspan transform. The presented solution is a generalization of the solutions found by Carrier et al. (2003) and Didenkulova and Pelinovsky (2011) for the case of a plane beach and a parabolic bay, respectively. The shoreline dynamics in U-shaped and V-shaped bays are computed via a double integral through standard integration techniques.

1. Introduction

Tsunamis pose danger to communities near coastal fjords and inlets [10,16,20] and understanding the tsunami run-up potential is crucial to saving lives and preventing damages. In many geophysical conditions, the tsunami generated by an earthquake can be considered as a long wave [22]. The first analytic solution to the long wave run-up problem was derived for a plane sloping beach [6]. To obtain this analytic solution, [6] introduced a new coordinate system in terms of the wave height and the bathymetry, and then defined a nonlinear coordinate transform now known as the classical Carrier–Greenspan (CG) transform, from cross-sectionally averaged shallow water wave equations into a linear variable velocity wave equation on the half line. The resulting wave equation was then solved through the use of a Hankel transform [5].

Later, Synolakis [21] expanded the work of Carrier and Greenspan to model wave run-up in a piecewise defined sloping plane beach formed by two different sloping regions. In 1994, Tadepalli and Synolakis considered the run-up of *N*-waves in a piecewise defined sloping plane beach [23]. This study resulted in run-up laws for *N*-waves. The run-up laws imply that a long wave with a leading depression has larger wave run-up than a similar long wave with a leading crest and also that the first wave run-up is not necessarily the largest one. By expanding the work of Carrier and Greenspan, Carrier et al. [7]

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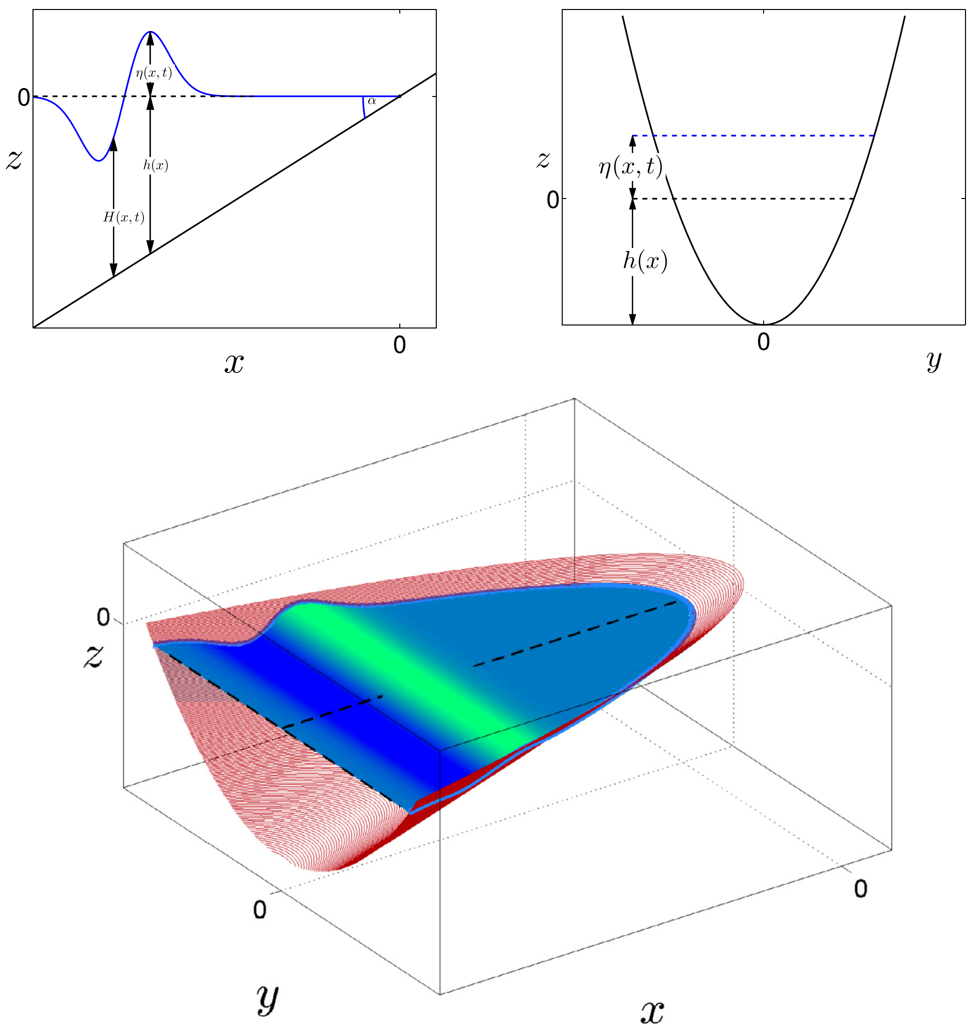

Fig. 1. Top left: a (x, z) cross-sectional view of an N-wave in a constantly sloping parabolic or $\mathcal{B}_2$ bay. Top right: a (y, z) cross-sectional view of a parabolic or $\mathcal{B}_2$ bay with water displacement $\eta(x)$. Bottom: a 3-D view of the bay and N-wave shown the two cross-sectional views above. Note that the bathymetry that is used is given by the function $z \sim |y|^2 + 0.01x$.

found a Green's function for the run-up problem with a non-zero initial velocity. This solution required the computation of highly singular elliptic integrals and used unnecessary linearizations of the CG transform. This drawback was addressed by Kanoglu and Synolakis [14] and a new Green's function that does not involve singular integrals was developed.

Until recently, no generalization of the CG transform for fjords and inlets existed. In 2011, the classical CG transformation was generalized by Didenkulova and Pelinovsky to the case of sloping bays with parabolic cross-sections [9] and then to sloping bays with U-shaped and V-shaped cross-sections [10]. Examples for U-shapes and V-shaped bays are shown in Figs. 1 and 2. For such bays, the water flow can be assumed to be uniform in the cross-section and thus the conservation of mass and linear momentum principles become

$$\frac{\partial S}{\partial t} + \frac{\partial}{\partial x}(uS) = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial H}{\partial x} = g\frac{dh}{dx}. \tag{2}$$

where $u = u(x, t)$ is the cross-sectional average velocity, $H = H(x, t)$ and $h = h(x)$ are, respectively, the total water depth and unperturbed water depth along the main axis of the bay, g is the acceleration due to gravity, and $S(x, t)$ is the cross-sectional area of the bay under water at the point (x, t). It is assumed that S is a function of total depth H only. We denote the water displacement as $\eta(x, t) = H(x, t) - h(x)$.

The generalizations of the CG transform for parabolic, U-shaped and V-shaped bays have led to the development of a d'Alembert solution for a parabolic bay of infinite length [10], an analytic solution for a parabolic bay of finite length [3], and an analytic solution for U-shaped and V-shaped bays of finite length [11].