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A meshless collocation approach with barycentric rational interpolation for two-dimensional hyperbolic telegraph equation

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ABSTRACT

In this paper, a meshless collocation method with barycentric rational interpolation is proposed to find and represent the solution of the two-dimensional hyperbolic telegraph equation. Barycentric rational interpolation functions are used for spatial variable and its partial derivatives, which produce a system of second order ordinary differential equations based on collocation method. The resulting system has been solved by central differential scheme. The accuracy and efficiency of proposed approach are demonstrated by several numerical experiments, where comparison is made with some earlier work. It is clear that the proposed method produces good results in comparison with those available in literature. Moreover, CPU time taken in our computation is much less. It is proved that the proposed method is very simple, fast and accurate.

1. Introduction

In the present work, we consider the following second-order linear two-space dimensional hyperbolic telegraph equation

$$u_{tt}(x,y,t)+2\alpha u_t(x,y,t)+\beta^2 u(x,y,t)=u_{xx}(x,y,t)+u_{yy}(x,y,t),\ (x,y,t)\in\Omega\times(0,T] \tag{1.1}$$

where $\Omega=[0,1]\times[0,1]$ is the problem region and $(0,T]$ is the time interval. u is an unknown function of x, y and t. α, β are the constant. For $\alpha>0$, $\beta=0$, Eq. (1.1) represents a damped wave equation and for $\alpha>\beta>0$ is called telegraph equation. The telegraph equation is an important equation of mathematics physics, which has been widely applied in many different fields such as transmission and propagation of electrical signals [1], random walk theory [2], and mechanical systems [3], etc.

The initial conditions are given by,

$$u(x,y,0)=\bar{u}(x,y),\ \ (x,y)\in\Omega \tag{1.2}$$

$$u_t(x,y,0)=\bar{v}(x,y),\ \ (x,y)\in\Omega \tag{1.3}$$

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while the Dirichlet boundary conditions are given as

$$\begin{cases} u(0,y,t) = h_1(y,t),\ (y,t) \in [0,1] \times [t > 0], & u(1,y,t) = h_2(y,t),\ (y,t) \in [0,1] \times [t > 0] \\ u(x,0,t) = h_3(x,t),\ (x,t) \in [0,1] \times [t > 0], & u(x,1,t) = h_4(x,t),\ (x,t) \in [0,1] \times [t > 0] \end{cases}, \tag{1.4}$$

Neumann boundary conditions are given as

$$\begin{cases} u_x(0,y,t) = g_1(y,t),\ (y,t) \in [0,1] \times [t > 0], & u_x(1,y,t) = g_2(y,t),\ (y,t) \in [0,1] \times [t > 0] \\ u_y(x,0,t) = g_3(x,t),\ (x,t) \in [0,1] \times [t > 0], & u_y(x,1,t) = g_4(x,t),\ (x,t) \in [0,1] \times [t > 0] \end{cases} \tag{1.5}$$

where Γ_u and Γ_t are non-intersection curves such that $\Gamma_u \cup \Gamma_t = \Gamma$, Γ is the closed curve bounding the region Ω.

Recently, increasing attention has been paid to the development, analysis, and implement of numerical solution for the hyperbolic telegraph equations. Mohanty et al. [4,5] proposed unconditionally stable ADI schemes for two and three space dimensional linear hyperbolic equations. Dehghan and Shokri [6] proposed a collocation scheme by using thin plate splines radial basis function. In [7], Dehghan and Mohebbi developed a high order compact finite difference and collocation scheme. In [8], two meshless methods were successfully applied for solving Eqs. (1.1)–(1.5). Author of [9] proposed the dual reciprocity boundary integral equation method. Xie et al. [10] presented two and three-level compact difference and alternating direction implicit schemes. In [11], the author studied a class of high accuracy scheme of $0(\tau^4 + h^2)$ and $0(\tau^4 + h^4)$ by using non-polynomial parameters cubic spline in space direction and compact difference in time direction. Difference quadrature method as a numerical discretization technique for the approximate of derivatives has been successfully applied to solve various partial differential equations such as telegraph equations [12–15], reaction–diffusion Brusselator equation [18], sine-Gordon equation [19] and hyperbolic equations [20,21]. A combination of boundary knot and analog equation method has been proposed by Dehghan and Salhi [16] for Eq. (1.1). Heydari et al. [17] developed a new approach of the Chebyshev wavelets method for solution of partial differential equations with boundary conditions of the telegraph type.

During the last two decades, meshless or meshfree methods have been developed and applied to solve partial differential equations (PDE). More and more researchers are devoting themselves to the study of meshless method, due to the fact that the construction of mesh is a tedious and difficult work in mesh-based method such as finite element method, finite difference method and boundary element method, etc [22]. According to the approximation used in derivation of the discrete mathematical models, the meshless methods can be largely categorized into two major groups, namely meshfree weak-form methods, such as the element-free Galerkin (EFG) method [23,31], reproducing kernel particle (RKPM) method [24,25], partition unity finite element (PUM) method [26], local radial point interpolation (LR-PIM) method [27], and meshfree local Petrove–Galerkin (MLPG) [28–30], etc., and meshless strong-form method, such as smoothing particle hydrodynamics (SPH) method [32], finite point method (FPM) [33], HP-meshless clouds method [34], and collocation method [35–38], etc. For the former, the weak forms are used to derive a set of algebraic equations through a numerical integration process using a set of integration domain that may be constructed globally or locally in the problem domain. The integration domain makes them computationally expensive, and not truly meshless. In the latter, governing equations and equations for boundary conditions are directly discretized at the scattered nodes to obtain a set of discretized system of equation by using the collocation techniques. Advantages of collocation methods are simple to implement, computationally efficient, and without any mesh for both field variable approximation and integration. Because of those advantages, meshless stong-form methods have been widely applied in the analysis for problems of fluid mechanics [22]. However, meshless strong-form methods are often unstable and less accurate, especially for the problems including Neumann boundary conditions. Some special techniques are designed to enforce Neumann boundary conditions. So, additional efforts are necessary in developing novel strong-form meshless method.

In most cases, the Lagrange approach is a choice for dealing with polynomial interpolations. The key is that the Lagrange polynomial must be manipulated through the formulas of barycentric interpolation [39]. The rational interpolation sometimes gives better approximations than polynomial interpolation, especially for large sequences, but it is difficult to control the poles. Happily, Float and Borman reported in [40] that there is in fact a whole family of barycentric rational interpolations which have no poles and arbitrarily high approximation orders. The family includes construction of Berrut [41] as a special case. The collocation method based on barycentric rational interpolation was recently extended to solve various ordinary and partial differential equations including 1D high order initial and boundary values problems [42] and non-linear Burgers' equation [43]. In fact, there are few reports about the application of barycentric rational interpolation in the literature, especially for the high dimensional problems. The interested reader can see [44–46].

This article presents a new numerical approach solving two-dimensional hyperbolic telegraph using the barycentric rational interpolation and collocation method. The advantages of proposed approach are very simple, fast and accurate. In addition, the boundary conditions including Neumann boundary conditions and Dirichlet boundary conditions can be implemented directly. The outline of the paper is as follows: In Section 2, the formula of two-dimensional barycentric rational interpolation is derived. In Section 3, the meshless collocation scheme with barycentric rational interpolation is presented. In Section 4, computational results for some test problems are illustrated and compared with some previous results and finally conclusions are included in Section 5.