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Efficient method for solving a system of nonlinear equations

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ABSTRACT

Various problems of pure and applied sciences can be studied in the unified frame work of the system of nonlinear equations. By using decomposition technique in conjunction with the coupled system of equations, we develop new iterative method for solving a system of nonlinear equations, which has better efficiency index. We prove the fifth-order convergence of the proposed method. We provide the comparison of efficiency index with some other well-known methods. For the implementation and performance of the new method, we solve the kinematic synthesis problem for steering, Chandrasekhar integral equation and combustion problem for a temperature of 3000 °C and compare the results with some existing methods.

1. Introduction

Consider the system of nonlinear equations of the type

$$\begin{aligned} f_1(x_1, x_2, \ldots, x_n) &= 0, \\ f_2(x_1, x_2, \ldots, x_n) &= 0, \\ &\cdots \\ f_n(x_1, x_2, \ldots, x_n) &= 0, \end{aligned}$$

where each function f_i, $i = 1, 2, \ldots, n$, maps a vector $\mathbf{X} = (x_1, x_2, \ldots, x_n)^t$ of the n-dimensional space $\mathbb{R}^n$ to the real line $\mathbb{R}$. The above system of n nonlinear equations in n unknowns can also be represented by defining a function $\mathbf{F}$ from $\mathbb{R}^n$ to $\mathbb{R}^n$ as

$$\mathbf{F}(\mathbf{X}) = (f_1(\mathbf{X}),\ f_2(\mathbf{X}), \ldots, f_n(\mathbf{X}))^t.$$

which we usually write $\mathbf{F}(\mathbf{X}) = \mathbf{0}$, where $\mathbf{F} : \mathbb{R}^n \to \mathbb{R}^n$ be nonlinear mapping from n-dimensional real linear space $\mathbb{R}^n$ into itself. The components f_i, $i = 1, 2, \ldots, n$, are the coordinate functions of $\mathbf{F}$.

A collection of nonlinear model problems have been presented by Moré [26], most of which are phrased in terms of system of nonlinear equations $\mathbf{F}(\mathbf{X}) = \mathbf{0}$. Grosan and Abraham [16] considered the various applications of the system of nonlinear equations in neurophysiology, chemical equilibrium problem, kinematics, combustion problem and economics modeling problem. Awawdeh [4] and Tsoulos and Stavrakoudis [45] solved the system of nonlinear equations by considering reactor and steering problems. Lin et al. [25] discussed the applications of system of nonlinear equations in the transport theory.

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To find the solution of the system of nonlinear equations, n-dimensional Newton method [41] is one of the fundamental tools and is quadratically convergent. Other different variants of the Newton method have been developed for solving $\mathbf{F}(\mathbf{X}) = \mathbf{0}$, using various techniques. Hueso et al. [20,21] have used Taylor polynomials for solving system of nonlinear equations. Babolian et al. [6], Darvishi and Barati [10] and Kaya and El-Sayed [23] have applied the Adomian decomposition technique [2,3] for solving the system of nonlinear equations. Abbasbandy [1], Darvishi and Barati [12] and Jafari and Daftardar-Gejji [22] used different modifications of the Adomian decomposition method for system of nonlinear equations. Özel [42] has used Noor's decomposition technique [28,31] to solve $\mathbf{F}(\mathbf{X}) = \mathbf{0}$. Golbabai and Javidi [14,15] applied the homotopy perturbation method [17] for solving $\mathbf{F}(\mathbf{X}) = \mathbf{0}$. Awawdeh [4] and Hosseini and Hosseini [19] have used the homotopy analysis method to solve systems of nonlinear equations. Babajee et al. [5], Darvishi and Barati [11,12], Frontini and Sormani [13] and Noor and Waseem [32,33] have applied quadrature formulas [7] to develop iterative methods for system of nonlinear equations.

In this paper, we use a new decomposition method to develop new iterative method for solving the system of nonlinear equations, which is quite different from the Adomian decomposition method [2,3]. In the implementation of the Adomian decomposition method, one has to calculate the derivatives of the so-called Adomian polynomials, which is itself a difficult problem. To overcome the drawback of Adomian decomposition method, we use the decomposition of Daftardar-Gejji and Jafari [8], as developed by Noor and Noor [30], to suggest several new iterative methods for solving the system of nonlinear equations. This new decomposition does not involve the high-order differentials of the function and is very simple as compared with the Adomian decomposition technique. He [18] has suggested that the system of nonlinear equations can be written as a coupled system of equations. This idea has been used by Noor [29], Noor et al. [34,35] to develop some iterative methods for solving nonlinear equations. In this paper, we use the decomposition technique by combining elegantly with the coupled system of equations, to develop new iterative method for solving system of nonlinear equations. We prove the fifth-order convergence of the proposed method. For the implementation and performance of the new method, we solve the kinematic synthesis problem for steering, Chandrasekhar integral equation and combustion problem for a temperature of 3000°C and compare the results with some existing methods.

2. Iterative methods

In this section, we develop the iterative methods for solving the system of nonlinear equations by using the decomposition technique.

Consider the system of nonlinear equations

$$\mathbf{F}(\mathbf{X}) = \mathbf{0}, \tag{1}$$

where $\mathbf{X} = (x_1, x_2, \dots, x_n)^t \in \mathbb{R}^n$. Assume that $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n) \in \mathbb{R}^n$ be a zero of the system of nonlinear equations (1), and $\boldsymbol{\Omega} = (\Omega_1, \Omega_2, \dots, \Omega_n) \in \mathbb{R}^n$ be an initial guess sufficiently close to $\boldsymbol{\alpha}$**.** By using the technique of Noor et al. [34,35], we can rewrite the system of nonlinear equations (1) as a coupled system of equations (see also [18]):

$$\mathbf{F}(\boldsymbol{\Omega}) + \mathbf{F}'(\boldsymbol{\Omega})(\mathbf{X} - \boldsymbol{\Omega}) + \mathbf{G}(\mathbf{X}) = \mathbf{0}, \tag{2}$$

$$\mathbf{G}(\mathbf{X}) = \mathbf{F}(\mathbf{X}) - \mathbf{F}(\boldsymbol{\Omega}) - \mathbf{F}'(\boldsymbol{\Omega})(\mathbf{X} - \boldsymbol{\Omega}), \tag{3}$$

where $\boldsymbol{\Omega}$ is the initial approximation for a zero of (1).

We can rewrite Eq. (2) in the following form:

$$\mathbf{X} = \boldsymbol{\Omega} - (\mathbf{F}'(\boldsymbol{\Omega}))^{-1}\mathbf{F}(\boldsymbol{\Omega}) - (\mathbf{F}'(\boldsymbol{\Omega}))^{-1}\mathbf{G}(\mathbf{X}) = \mathbf{C} + \mathbf{N}(\mathbf{X}), \tag{4}$$

where

$$\mathbf{C} = \boldsymbol{\Omega} - (\mathbf{F}'(\boldsymbol{\Omega}))^{-1}\mathbf{F}(\boldsymbol{\Omega}), \tag{5}$$

and

$$\mathbf{N}(\mathbf{X}) = -(\mathbf{F}'(\boldsymbol{\Omega}))^{-1}\mathbf{G}(\mathbf{X}), \tag{6}$$

where $\mathbf{N}(\mathbf{X})$ is a nonlinear function.

Now, we use a new decomposition technique, which is mainly due to Daftardar-Gejji and Jafari [8], to construct a sequence of high-order iterative methods for the system of nonlinear equations. This decomposition of nonlinear operator $\mathbf{N}(\mathbf{X})$ is quite different than that of the Adomian decomposition [2,3].

The main idea of this technique is to look for a solution of Eq. (4) having the series form

$$\mathbf{X} = \sum_{i=0}^{\infty} \mathbf{X}_i. \tag{7}$$

The nonlinear operator $\mathbf{N}$ can be decomposed as

$$\mathbf{N}\left(\sum_{i=0}^{\infty} \mathbf{X}_i\right) = \mathbf{N}(\mathbf{X}_0) + \sum_{i=1}^{\infty} \left\{ \mathbf{N}\left(\sum_{j=0}^{i} \mathbf{X}_j\right) - \mathbf{N}\left(\sum_{j=0}^{i-1} \mathbf{X}_j\right) \right\}. \tag{8}$$