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Existence of solution for some nonlinear two-dimensional Volterra integral equations via measures of noncompactness

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ABSTRACT

In this paper, we analyze the existence of solution for two-dimensional nonlinear Volterra integral equations (2DVIE) by using the techniques of measures of noncompactness and Petryshyn fixed point theorem which contains as particular cases a lot of integral and functional-integral equations that arise in nonlinear analysis. Also some illustrative examples are given.

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1. Introduction

The functional integral equations describe many physical phenomena in various areas of natural science, mathematical physics, mechanics and population dynamics [4,5,8,10,14]. Recently, there have been several successful attempt to apply the concept of measure of noncompactness in the study of the existence and behavior of solutions of nonlinear integral equations [1,2,6,9,13]. In this paper, we present and prove a new existence theorem for solution of two dimension nonlinear integral equations which are formulated in terms of condensing operators in Banach spaces

$$u(s,t)=q\left(s,t,u(s,t),\int_0^s h(s,t,\zeta,u(\zeta,t))d\zeta,\int_0^s\int_0^t k(s,t,x,y,u(x,y))dydx\right), \tag{1.1}$$

where, $(s,t)\in[0,a]\times[0,b]$ for suitable functions u, q, h, k that introduced below.

The main goal of this study is to investigate existence of solution Eq. (1.1). For this purpose, we use a fixed theorem due to Petryshyn [15] that has been discussed as a generalization of Darbo's fixed theorem [3]. Numerous authors have made successful efforts to solve many functional integral equations by powerful tools of Darbo's condition [1,2,6,9,12,13]. In our consideration we use the Petryshyn condition for solving Eq. (1.1). The structure of this paper is as follows. In Section 2, we introduce some preliminaries and use them to obtain our main results. Section 3 is devoted to state and prove existence theorem for equations involving condensing operators using the fixed point theorem. Finally in Section 4, we offer some examples that verify the application of this kind of non-linear functional-integral equations.

2. Preliminaries

In this section, we introduce notations, definitions and preliminaries facts which are used throughout this paper. Let X be a Banach space . We write $\bar{B}_r=\{x\in X:\|x\|\le r\}$ for the closed ball and $\partial\bar{B}_r=\{x\in X:\|x\|=r\}$ for the sphere in X around 0 with radius $r>0$.

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Measures of non compactness are very useful tools in functional analysis, for instance in metric fixed point theory and in the theory of operator equations in Banach spaces. The first measure of non compactness, denoted by α, was defined and studied by Kuratowski in 1930.

Definition 2.1 [11]**.** The Kuratowski measure of noncompactness (or set measure of noncompactness)

$$\alpha(M) = \inf\{\sigma > 0 : M \text{ may be covered by finitely many sets of diameter} \leq \sigma\}. \tag{2.1}$$

Other measures of noncompactness were introduced by Gol'denšteĭn.

Definition 2.2 [7]**.** The Hausdorff (or ball) measure of noncompactness

$$\psi(M) = \inf\{\sigma > 0 : \text{there exists a finite } \sigma\text{-net for } M \text{ in } X\}, \tag{2.2}$$

where by a finite σ-net for M in X we mean, as usual, a set $\{z_1, z_2, \dots, z_m\} \subset X$ such that the balls $B_\sigma(X; z_1), B_\sigma(X; z_2), \dots, B_\sigma(X; z_m)$ over M. These measures of noncompactness are mutually equivalent in the sense that

$$\psi(M) \leq \alpha(M) \leq 2\psi(M)$$

for any bounded set $M \subset X$.

It is easy to see that the following basic results hold for any measure of noncompactness.

Theorem 2.1 [15]**.** *Let X be a Banach space, $\lambda \in \mathbb{R}$ and $M, N \subset X$ bounded. Then*

(i) $\psi(M \cup N) = \max\{\psi(M), \psi(N)\}$;
(ii) $\psi(M + N) \leq \psi(M) + \psi(N)$;
(iii) $\psi(\lambda M) = |\lambda| \psi(M)$;
(iv) $\psi(M) \leq \psi(N)$ *for* $M \subseteq N$;
(v) $\psi(\bar{co}M) = \psi(M)$;
(vi) $\psi(M) = 0$ *if and only if M is precompact.*

In what follows, we will work in the space $C[0, a] \times [0, b]$ consisting of all real functions defined and continuous on the interval $[0, a] \times [0, b]$. The space $C[0, a] \times [0, b]$ is equipped with the standard norm

$$\|x\| = \sup\{|x(s, t)| : s \in [0, a], t \in [0, b]\}.$$

Recall that the modulus of continuity of a function $u \in C[0, a] \times [0, b]$ is defined by

$$\omega(u, \sigma) = \sup\{|u(x, y) - u(s, t)| : |x - s|, |y - t| \leq \sigma\}.$$

We have then $w(u, \sigma) \to 0$, as $\sigma \to 0$, since u is uniformly continuous on $[0, a] \times [0, b]$. More generally, if this limit relation holds uniformly for u running over some bounded set $M \subset C$, then M is equicontinuous, and vice versa. Therefore the following result is not too surprising:

Theorem 2.2. *On the space $C[0, a] \times [0, b]$, the measures of noncompactness (2.2) is equivalent to*

$$\psi(M) = \lim_{\sigma \to 0} \sup_{u \in M} \omega(u, \sigma) \tag{2.3}$$

for all bounded sets $M \subset C[0, a] \times [0, b]$.

For our purpose we use (2.3) in the rest of the paper. Closely associated with the measures of noncompactness is the concept of k-set contraction.

Definition 2.3. Let T: $X \to X$ be a continuous mapping of a Banach space X. T is called a k-set contraction if for all $A \subset X$ with A bounded, $T(A)$ is bounded and $\alpha(TA) \leq k\alpha(A)$, $0 < k < 1$.

If

$$\alpha(TA) < \alpha(A), \text{ for all } \alpha(A) > 0,$$

then T is called densifying or condensing map [16].

A k-set contraction with $k \in (0, 1)$, is densifying, but the converse is not true.

Now we state Petryshyn fixed point theorems of Petryshyn [15] which are used in the main results .

Theorem 2.3 [15], see also [17]**.** *Let $\bar{B}_r$ be an open ball about the origin in a Banach space X. If $T : \bar{B}_r \to X$ is a densifying mapping that satisfies the boundary condition,*

$$\text{If } T(x) = kx, \text{ for some x in } \partial B_r \text{ then } k \leq 1, \tag{P}$$

then $F(T)$, the set of fixed points of T in $\bar{B}_r$ is nonempty.

This property allows us to characterize solution of the integral Eq. (1.1) and will be used in the next section.

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