Applied Mathematics and Computation 275 (2016) 268–286

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

On the evaluation of finite-time ruin probabilities in a dependent risk model

Dimitrina S. Dimitrova*, Vladimir K. Kaishev, Shouqi Zhao

Faculty of Actuarial Science and Insurance, Cass Business School, City University London, 106 Bunhill Row, EC1Y 8TZ London, UK

ARTICLE INFO

Keywords:
Finite time ruin probability
Dependent risk modeling
Appell polynomials
Numerical implementation
Order statistics

ABSTRACT

This paper establishes some enlightening connections between the explicit formulas of the finite-time ruin probability obtained by Ignatovand Kaishev (2000, 2004) and Ignatov et al. (2001) for a risk model allowing dependence. The numerical properties of these formulas are investigated and efficient algorithms for computing ruin probability with prescribed accuracy are presented. Extensive numerical comparisons and examples are provided.

1. Introduction

Research on ruin probability beyond the classical risk model has intensified in recent years. More general ruin probability models assuming dependence between either claim amounts or claim arrivals, or cross-dependence between both arrivals and sizes of claims, and non-linear aggregate premium income have been considered in the actuarial and applied probability literature. Such models are better suited to reflect the dependence in the arrival and severity of losses generated by portfolios of insurance policies. Exploring ruin probability theoretically and numerically, under these more general dependence assumptions, is of utmost importance within the Solvency II framework of internal insolvency-risk model building.

Albrecher and Boxma [2,3] have considered a collective infinite-horizon ruin model of (semi-)Markovian type where the dependence structure assures that both the consecutive claim inter-arrival times and claim sizes are respectively correlated, and there could also be a cross-correlation between them, and, as in the classical case, premiums accumulate linearly in time. The model considered by Albrecher and Boxma [3] is reasonably general and embeds the classical compound Poisson, and the Sparre-Andersen model with phase type distributed claim inter-arrival times as special cases. However, as the authors note, “in concrete cases, it is sometimes not possible to evaluate the occurring expressions”. It has to be noted also that these expressions relate to the infinite-time ruin case which is not particularly relevant to finite-time applications such as modeling insurance solvency.

Under the classical constant premium rate assumption, Albrecher and Teugels [1] consider a random walk model in which the waiting time for a claim and the claim size are dependent. Asymptotic exponential estimates for both finite and infinite time ruin probabilities are then obtained for light-tail claims, using the Laplace transform. Boudreault et al. [7] also assume that the current claim (amount) is dependent on the inter-occurrence time preceding it, and more precisely that the corresponding conditional density is defined as a mixture of two arbitrary densities with weights defined by exponentials whose powers are proportional to the preceding inter-occurrence time. In [8] the dependence between the claim amount and its corresponding inter-arrival time is modeled by a generalized Farlie–Gumbel–Morgenstern copula. The Laplace transform of the Gerber–Shiu discounted penalty function is derived, and for exponential claims, an explicit formula for the Laplace transform of the ruin time

* Corresponding author. Tel.: +44 2070408470.
E-mail address: d.dimitrova@city.ac.uk (D.S. Dimitrova).

http://dx.doi.org/10.1016/j.amc.2015.11.082

is provided. In a recent paper [26] consider dependence in a risk model under the assumption that the claims arrive according to an order statistics process.

A collective finite-horizon ruin probability model with Poisson claim arrivals, dependent discrete claim amounts having any joint distribution but independent of the claim arrival times, and aggregate premium income represented by any non-decreasing positive, real valued function, has been considered by Ignatov and Kaishev [13]. They give an explicit finite-horizon ruin probability formula in terms of infinite sums of determinants which are shown by the authors to admit representation as classical Appell polynomials. Some useful properties of Appell polynomials, including a recurrence formula are given in the Appendix of that paper. An improved explicit and exact version of the ruin probability formula of Ignatov and Kaishev [13], involving finite summation, is given in [15]. In [14], the same ruin model is considered but assuming the claim amounts have arbitrary continuous (possibly dependent) joint distribution. The finite-time ruin probability formula in that case is obtained explicitly in terms of classical Appell polynomials.

Our goal in this paper is two-fold. First, we summarize the explicit ruin probability formulas which appear in the papers by Ignatov and Kaishev [13,14] and Ignatov et al. [15], deduce new alternative expressions, and establish some enlightening connections between these formulas. The latter allow for a unified treatment and a fair comparison of their numerical efficiency. Thus, we also study the numerical properties of these formulas and propose an algorithm for their efficient evaluation with a preliminary prescribed accuracy. Based on a series of examples, we demonstrate that these formulas are useful not only theoretically but also for computing ruin probabilities in various risk models with dependence. The latter is important in practical applications. For example, as recently pointed out by Das and Kratz [9], the need to evaluate the Ignatov–Kaishev ruin probability formulas naturally arises in the context of designing early warning systems against ruin of insurance companies. This need also arises in the context of reserving and risk capital allocation in particular, for operational risk, see [17].

This paper is organized as follows. In Section 2, we introduce our main model and give the formulas obtained by Ignatov et al. [15] and Ignatov and Kaishev [14] for both discrete and continuous claim amounts, and also demonstrate the interconnection between these formulas. The latter incorporates classical Appell polynomials and thus, Section 3 introduces various recurrence expressions for computing classical Appell polynomials. Section 4 provides a method of computing survival probability with a prescribed accuracy and a simulation method employing order statistics proposed by Dimitrova and Kaishev [10] is introduced in Section 5. In Section 6, we study the numerical properties of all theoretical results and provide several numerical examples for both discrete and continuous, dependent and independent claim severities. Section 7 concludes the paper.

2. On non-ruin probability formulas and relations between them

Let us first recall the model which we will be concerned with, which has first been considered in [13–15]. Let the random variables $W_1, W_2, \dots,$ denote claim severities, and let $Y_1, Y_2, \dots,$ denote their partial sums, i.e. $Y_1 = W_1, Y_2 = W_1 + W_2, \dots,$. If claim severities $W_1, W_2, \dots, W_k$ are considered continuous random variables, then $\psi(w_1, \dots, w_k)$ will denote their joint density and $f(y_1, \dots, y_k)$ will denote the joint density of $Y_1, Y_2, \dots, Y_k$. Clearly, $\psi(w_1, \dots, w_k) = f(w_1, w_1 + w_2, \dots, w_1 + \dots + w_k)$ and $f(y_1, \dots, y_k) = \psi(y_1, y_2 - y_1, \dots, y_k - y_{k-1})$. In the case of discrete claim severities $W_1, W_2, \dots, W_k$, their joint probability mass function $P(W_1 = w_1, \dots, W_k = w_k)$ is denoted by $p(w_1, \dots, w_k)$.

Let $\tau_1, \tau_2, \dots,$ denote the claim inter-arrival times assumed to be independent, identically distributed random variables, following an exponential distribution with mean $1/\lambda$, i.e. $\tau_i \sim Exp(\lambda),\ i = 1, 2, \dots,$. Thus, the number of claims up to time t is modeled by the Poisson process $N_t = \max\{i : \tau_1 + \dots + \tau_i \le t\},\ t > 0$. We denote by $T_1, T_2, \dots,$ the arrival times of consecutive claims, i.e. $T_i = \tau_1 + \dots + \tau_i,\ i = 1, 2, \dots,$. Let $h(t)$ denote the premium income function of an insurance company, which is assumed a non-negative and non-decreasing real valued function defined on $\mathbb{R}_+$. It is worth noting that the condition $\lim_{t\to\infty} h(t) = +\infty$ is not necessarily required since we are interested in finite-time ruin probability. Let us also note that the function $h(t)$ does not need to be necessarily continuous. If it is discontinuous, we define $h^{-1}(y) = \inf\{z : h(z) \ge y\}$. The insurance company's surplus process is expressed as $R_t = h(t) - S_t$, where $S_t = Y_{N_t}$ is the aggregate claim amount process, and the instant of ruin T is defined as

$$T := \inf\{t : t > 0,\ R_t < 0\}$$

or $T = \infty$ if $R_t >= 0$ for all t. Under this reasonably general risk model, an exact formula for the probability of non-ruin within a finite time interval $[0, x]$, $P(T > x)$, assuming discrete claim severities, has been given in [15], based on the formula derived in [13]. The former can easily be expressed as

$$P(T > x) = e^{-\lambda x} \sum_{k=1}^{n+1} \left(\sum_{w_1=1}^{n-(k-2)} \sum_{w_2=1}^{n-(k-3)-w_1} \cdots \sum_{w_{k-1}=1}^{n-w_1-\dots-w_{k-2}} \sum_{w_k=n+1-w_1-\dots-w_{k-1}}^{\infty} p(w_1, \dots, w_k) \right.$$
$$\left. \times \sum_{j=0}^{k-1} (-1)^j b_j(\nu_1, \dots, \nu_j) \lambda^j \sum_{m=0}^{k-j-1} \frac{(\lambda x)^m}{m!} \right). \tag{1}$$

where n is the integer part of $h(x)$, i.e. $n = \lfloor h(x) \rfloor$, $\nu_k = h^{-1}(w_1 + \dots + w_k)$, and $b_j(\nu_1, \dots, \nu_j)$ is defined recurrently as

$$b_j(\nu_1, \dots, \nu_j) = (-1)^{j+1} \frac{\nu_j^j}{j!} + (-1)^{j+2} \frac{\nu_j^{j-1}}{(j-1)!} b_1(\nu_1) + \dots + (-1)^{j+j} \frac{\nu_j^1}{1!} b_{j-1}(\nu_1, \dots, \nu_{j-1}) \tag{2}$$