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Coupling strength computation for chaotic synchronization of complex networks with multi-scroll attractors

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ABSTRACT

In this paper synchronization of *N*-coupled chaotic oscillators with multi-scroll attractors is presented. *N* chaotic oscillators are coupled in regular and irregular topologies. The generalizations of the Genesio & Tesi and Chua's chaotic oscillators are used as generators of multi-scroll attractors. An alternative scheme for computing the coupling strength is proposed. Synchronization is achieved through the coupling matrix and by using the resulting alternative values. In general, the range of values obtained with the proposed method is smaller than the one given by Wang & Chen method. The effectiveness of this coupling strength is verified through numerical simulations.

1. Introduction

The word synchronization describes the time correspondence of different processes [1,2]; as an action, it describes the time match of two or more phenomena.

In the last two decades, chaotic synchronization has received increasing attention after the result of Pecora and Carroll. In 1990, Pecora and Carroll synchronized two identical chaotic oscillators with different initial conditions for the first time [3]. Chaotic synchronization has been intensively studied since then and many methods have been proposed to achieve synchronization between two chaotic oscillators. Some of those methods are: chaotic synchronization through adaptive control and observer [4,5], output synchronization problem [6–9], synchronization through back-stepping design and adaptive feedback injections [10,11], synchronization using sliding mode controllers [12–14], synchronization through active control method [15–17], synchronization with information exchanges at discrete-time [18–20], synchronization through filtering techniques and Hamiltonian forms [21,22] for instance.

Nowadays, one of the most important challenges, to achieve synchronization in a complex network, is to obtain suitable values for the coupling strength. For that reason, efforts in this article are devoted to calculating an alternative coupling strength. The objective is to develop a novel method to obtain a coupling strength with minimally invasive value to ensure the network's synchrony.

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For that purpose, the generalizations of the Genesio & Tesi and Chua's chaotic oscillators will be used. These models are multi-scroll attractor generators.

In 1992, the Genesi & Tesi system was developed to examine the harmonic balance method and determine the existence and location of the chaotic behavior [23,24]. Genesio and Tesi successfully applied the method and proved that the model exhibited chaos [23,24]. A generalization of the original Genesio & Tesi system [24], to generate n-scroll was reported in [25]. The family of chaotic oscillators, called *grid scroll attractors*, increase the amount of scrolls along any of their variables, so they can be in 1, 2, 3-D.

By modifying some parameters, the generalized Chua's chaotic oscillator allows us to increase the number of scrolls. This oscillator is a generalization of Chua's original model and it is based on amending the system's nonlinearity. This is made by inserting additional breaking points, which results in an increase of the scrolls in the attractor.

In this paper, authors propose computing the coupling strength depending on the stability of the synchronization error system, which results in a novel and easier way to achieve synchrony in the complex network. The resulting coupling strength will be smaller than the one given by the condition of Wang and Chen Theorem [26] which shows poor effectiveness for irregular topologies. For clarity of the manuscript, the proposed method, which can be considered as the main contribution of this work, is developed in the appendix.

This paper is organized as follows: Section 2 describes a brief review on complex dynamical networks and their synchronization. In Section 3, the multi-scroll attractor generators Genesio & Tesi 3-D chaotic oscillator and generalized Chua's chaotic oscillator are described. The synchronization of N-coupled chaotic oscillators in regular and irregular networks and the corresponding simulation results are provided in Section 4. In Section 5, some conclusions are given. In Appendix A the procedure proposed to obtain the coupling strength is given.

2. Complex networks

Among the available definitions of complex network, in the present case we refer to the definition suggested by Wang [27].

Definition 1. A complex network is defined as an interconnected set of oscillators (two or more), where each oscillator is a fundamental unit, with its dynamic depending of the nature of the network.

Each oscillator is defined as follows:

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + \mathbf{u}_i, \quad \mathbf{x}_i(0), \quad i = 1, \quad 2, \ldots, \quad N, \tag{1}$$

where N is the network's size, $\mathbf{x}_i = [x_{i1}\ x_{i2}\ \ldots\ x_{in}] \in \Re^n$ represents the state variables of the ith oscillator. $\mathbf{x}_i(0) \in \Re^n$ are the initial conditions for ith oscillator. $\mathbf{u}_i \in \Re^n$ establishes the synchronization between two or more oscillators and is defined as follows [26]:

$$\mathbf{u}_i = c\sum_{j=1}^{N} a_{ij}\mathbf{\Gamma}\mathbf{x}_j, \quad i = 1, \quad 2, \ldots, \quad N, \tag{2}$$

the constant $c > 0$ represents the coupling strength. $\Gamma \in \Re^{n\times n}$ is a constant matrix to determine the coupled state variable of each oscillator. Assume that $\Gamma = \mathrm{diag}(r_1, r_2, \ldots, r_n)$ is a diagonal matrix. If two oscillators are linked through their kth state variables, then, the diagonal element $r_k = 1$ for a particular k and $r_j = 0$ for $j \neq k$.

The matrix $\mathbf{A} \in \Re^{N\times N}$ with elements a_{ij} is the coupling matrix which shows the connections between oscillators, if the oscillator ith is connected to the oscillator jth, then $a_{ij} = 1$, otherwise $a_{ij} = 0$ for $i \neq j$. The diagonal elements of matrix $\mathbf{A}$ are defined as

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij} = -\sum_{j=1, j\neq i}^{N} a_{ji} \quad i = 1, 2, \ldots, N. \tag{3}$$

The dynamical complex network (1) and (2) achieves synchronization if $\mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots = \mathbf{x}_N(t)$, as $t \to \infty$, which means that Eq. (4) holds

$$\lim_{t\to\infty} \|\mathbf{x}_i(t) - \mathbf{x}_j(t)\| = 0, \quad i = 1, \ldots, N-1;\ j = i+1, \ldots, N. \tag{4}$$

Complex networks with N identical oscillators are considered. Each oscillator can be either a multi-scroll attractor Genesio & Tesi 3-D chaotic oscillator or the generalized Chua's chaotic oscillator. Regular and irregular topologies such as the ones shown in Fig. 1 are considered.

3. Multi-scroll chaotic oscillators

In this section the multi-scroll chaotic oscillator Genesio & Tesi 3-D and the generalized Chua's chaotic oscillator will be described. These multi-scroll attractor generators will be used to compose the complex networks.