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Exponential stability of time-delay systems via new weighted integral inequalities

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ABSTRACT

In this paper, new weighted integral inequalities (WIIs) are first derived based on Jensen's integral inequalities in single and double forms. It is theoretically shown that the newly derived inequalities in this paper encompass both the Jensen inequality and its most recent improvement based on Wirtinger's integral inequality. The potential capability of WIIs is demonstrated through applications to exponential stability analysis of some classes of time-delay systems in the framework of linear matrix inequalities (LMIs). The effectiveness and least conservativeness of the derived stability conditions using WIIs are shown by various numerical examples.

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1. Introduction

It is well-known that time-delay is frequently encountered in various practical engineering systems and usually is a source of poor performance, oscillations or instability [1,2]. Therefore, the problem of stability analysis and its applications to control of time-delay systems is essential and of great importance for both theoretical and practical reasons [1] which has attracted considerable attention during the last decade (see, for example, [3–8] and the references therein). Many important results on asymptotic stability of time-delay systems have been established using the Lyapunov–Krasovskii functional (LKF) method in the framework of linear matrix inequalities (LMIs) [9–12].

While the problem of asymptotic stability of time-delay systems has been well studied and developed, the problem of exponential stability is also very important due to the facts that, on one hand, asymptotic stability is a synonym of exponential stability [13], on the other hand, in many applications, it is important to determine the convergence rate of system states or to find estimates of the transient decaying rate of the system [14–16]. A great deal of efforts has been devoted to study exponential stability of time-delay systems recently [17–29]. To derive an estimate, also referred to α-stability, of exponential convergence rate of such a time-delay system, various approaches have been proposed in the literature. For example, the authors of [14,17–20] used the state transformation $\xi(t) = e^{\alpha t}x(t)$ combining with the Lyapunov–Krasovskii functional method to derive exponential stability conditions. In [19,21], improved stability conditions were proposed by using model transformations. A widely used approach based on modified LKFs with exponential weighted functions was used in [13,22–29] to derive LMIs conditions ensuring exponential stability of the system. Some other methods such as estimating the Lyapunov components or modified comparison principle were proposed in [5,16,30,31].

However, looking at the literature, it can be realized that the proposed methods in most of the aforementioned works usually introduce conservatism in exponential stability conditions not only on the exponential convergence rate but also on the maximal

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allowable delay and the number of matrix variables. Therefore, aiming at reducing conservativeness of exponential stability conditions in the framework of the Lyapunov–Krasovskii functional method, an important and relevant issue is to improve some integral-based inequalities [9,12].

Motivated by the above discussion, our main goal in this paper is to derive some new weighted integral inequalities (WIIs) which are suitable to use in combination with the Lyapunov–Krasovskii functional method to derive improved and less conservative exponential stability conditions for time-delay systems. To tackle with this problem, we first extend the celebrated Jensen inequality [32] to a weighted integral version. We then introduce a constructive approach inspired from a recent work [11] to derive new WIIs. It is also pointed out that the newly derived inequalities in this paper encompass the Jensen inequality and its recent improvements based on Wirtinger's integral inequalities [9,10] as some critical cases. The potential capability of WIIs is shown through applications to derive new exponential stability conditions for some classes of time-delay systems in the framework of linear matrix inequalities.

The remaining of this paper is organized as follows. In Section 2, some preliminary results are presented. New weighted integral inequalities and their applications in exponential stability analysis of some classes of time-delay systems are presented in Section 3 and 4, respectively. Section 5 provides numerical examples including a practical example to demonstrate the effectiveness of the obtained results. The paper ends with a conclusion and references.

2. Preliminaries

It can be realized in many contributions that, to derive exponential estimates for time-delay systems, a widely used approach is the use of weighted exponential Lyapunov–Krasovskii functional [13]. For example, a functional of the form

$$V(x_t) = \int_{-\tau}^{0}\int_{t+s}^{t} e^{\alpha(u-t)}\dot{x}^T(u)R\dot{x}(u)duds \tag{2.1}$$

where x is the state vector, scalars $\alpha > 0$, $\tau > 0$ and matrix $R > 0$, has been used in many works in the literature [22,24–27]. The derivative of $V(x_t)$ is given by

$$\dot{V}(x_t) = \tau\dot{x}^T(t)R\dot{x}(t) - \int_{t-\tau}^{t} e^{\alpha(s-t)}\dot{x}^T(s)R\dot{x}(s)ds. \tag{2.2}$$

In order to generate LMIs conditions, an estimate on the second term of (2.2) is obviously needed. The problem raised here is how to find a tighter lower bound of a weighted integral of quadratic terms in the following form

$$I_w(\varphi,\alpha) = \int_a^b e^{\alpha(s-b)}\varphi^T(s)R\varphi(s)ds$$

where $\alpha > 0$ is a scalar, $\varphi \in C([a,b],\mathbb{R}^n)$ and R is a symmetric positive definite matrix. For a special case, when $\alpha = 0$ then we write $I(\varphi)$ instead of $I_w(\varphi, 0)$.

Inspired by the proof of Jensen's inequality [32], we have the following results which in this paper we refer to as weighted integral inequalities (WIIs) in single and double forms.

Lemma 2.1. *For a given $n\times n$ matrix $R > 0$, scalars $b > a$, $\alpha > 0$ and a function $\varphi \in C([a,b],\mathbb{R}^n)$, the following inequalities hold*

$$\int_a^b e^{\alpha(s-b)}\varphi^T(s)R\varphi(s)ds \geq \frac{\alpha}{\gamma_0}\left(\int_a^b \varphi(s)ds\right)^T R\left(\int_a^b \varphi(s)ds\right), \tag{2.3}$$

$$\int_a^b\int_s^b e^{\alpha(u-b)}\varphi^T(u)R\varphi(u)duds \geq \frac{\alpha^2}{\gamma_1}\left(\int_a^b\int_s^b \varphi(u)duds\right)^T R\left(\int_a^b\int_s^b \varphi(u)duds\right), \tag{2.4}$$

where γ_k, $k \geq 0$, denotes the residual $e^{\alpha(b-a)} - \sum_{j=0}^{k}\frac{\alpha^j(b-a)^j}{j!}$.

Proof. By taking integral of the inequality $\begin{bmatrix} e^{\alpha(s-b)}\varphi^T(s)R\varphi(s) & \varphi^T(s) \\ \varphi(s) & e^{\alpha(b-s)}R^{-1}\end{bmatrix} \geq 0$ we obtain

$$\begin{bmatrix} I_w(\varphi,\alpha) & \int_a^b \varphi^T(s)ds \\ \int_a^b \varphi(s)ds & \rho(\alpha)R^{-1}\end{bmatrix} \geq 0$$

which implies (2.3) by the Schur complement. The proof of (2.4) is similar and, thus, is omitted here. □

Remark 2.1. In order to seek exponential stability conditions for time-varying delay systems, a widely used approach in the literature (see, for example, [15,23–25,28] and the references therein) is that the integral term $I_w(\varphi,\alpha)$ is first estimated as $I_w(\varphi,\alpha) \geq e^{-\alpha(b-a)}I(\varphi)$, then the celebrated Jensen integral inequality and its variants are used to manipulate $I(\varphi)$. It is worth to point out that the estimate $\frac{\alpha}{\gamma_0} > e^{-\alpha(b-a)}$ holds for all $\alpha > 0$, $b > a$. Therefore, (2.3) gives a tighter lower bound in comparison to the common estimate $I_w(\varphi,\alpha) \geq e^{-\alpha(b-a)}I(\varphi)$.

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