Applied Mathematics and Computation 275 (2016) 361–371

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# Robust quantized $H_\infty$ filtering for discrete-time uncertain systems with packet dropouts[☆]

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ARTICLE INFO

*Keywords:*
Robust $H_\infty$ filtering
Measurement quantization
Packet dropouts
Linear matrix inequalities

ABSTRACT

This paper considers the problem of $H_\infty$ filtering for uncertain discrete-time systems with quantized measurements and packet dropouts. The time-invariant uncertain parameters are supposed to reside in a polytope. The system measurement outputs are quantized by a memoryless logarithmic quantizer before being transmitted to the filter and the performance of packet dropouts is described by Bernoulli random binary distribution. Attention is focused on the design of $H_\infty$ filter to mitigate the effects of quantization and packet dropouts, which ensured not only stochastically stability but also a prescribed $H_\infty$ noise attenuation level. Via parameter-dependent Lyapunov function approach and introducing some slack variables, sufficient conditions for the existence of an $H_\infty$ filter are expressed in terms of linear matrix inequalities (LMIs). Two examples are provided to demonstrate the effectiveness and applicability of the proposed method.



## 1. Introduction

As one of the most important problem in signal processing and control theory, filtering, especially $H_\infty$ filtering, has attracted a recurring interest [1,2]. This is mainly due to $H_\infty$ filtering is more general than classical Kalman filtering. Compared with classical Kalman filtering, $H_\infty$ filtering has obvious advantages, that is, no statistical assumption on the noise signals is needed and more robust than Kalman filtering.

The study on robust $H_\infty$ filtering for polytopic uncertain systems has achieved many important advances over the past decades. Earlier results for robust $H_\infty$ filtering are based on quadratic Lyapunov functions. Though the quadratic Lyapunov function method greatly facilitates the analysis and synthesis of robust $H_\infty$ filtering for polytopic uncertain systems, the results obtained within quadratic Lyapunov function can be conservative due to the fact that a single Lyapunov matrix is used to the entire uncertainty domain. In recent studies, parameter-dependent Lyapunov function approach has been introduced to deal with robust $H_\infty$ filtering of systems with polytopic uncertainties, see for instance [3–7]. A comprehensive study on robust $H_2$ and $H_\infty$ filtering for discrete-time systems with polytopic uncertainties has been given in [3]. Following this work, by introducing a new slack matrix, less conservative conditions of robust $H_2$ and $H_\infty$ filtering are proposed in [4] for both discrete and

☆ This work was supported in part by the National Natural Science Foundation of China under grant no. 61104071, the Program for Liaoning Excellent Talents in University, China under grant no. LJQ2012095, the Open Program of the Key Laboratory of Manufacturing Industrial Integrated Automation, Shenyang University, China under grant no. 1120211415, the Natural Science Foundation of Liaoning Province of China under grant no. 2015020052.

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http://dx.doi.org/10.1016/j.amc.2015.11.069
0096-3003/

continuous-time systems with polytopic uncertainties. By using Fisher lemma, more slack matrixes are introduced and optimization results are given in [5]. Subsequently, by using Finsler lemma and Projection lemma better results of robust $H_\infty$ filtering for discrete-time systems with polytopic uncertainties are reported in [6]. Recently, by considering auxiliary slack variables with free structure and introducing additional scalar parameter, new conditions of $H_\infty$ filter design for discrete-time systems with polytopic uncertainties have been proposed in [7]. Some new results about $H_\infty$ filtering see [8,9].

It is worth pointing that the above literature of robust $H_\infty$ filtering the communication channel is assumed to be ideal that means the effects of quantization and packet dropouts have not been considered. However, this may not be true in actual application, especially in the networked control systems (NCSs). On the other hand, Kalman [10] and Dong et al. [11] pointed that quantization and packet dropouts may be caused unfavorable influence for control performance and system stability, respectively. Follow these works, considerable efforts have been devoted for better analysis and design of systems with quantization [12–15] and packet dropouts [16–21]. Recently, many researchers have focused on the design of filter for different systems with mentioned above phenomenona. Noticeable works include [22–28]. In [22], the problem of $H_\infty$ filtering for linear discrete-time systems with quantization has been investigated. A study on $H_\infty$ filtering for continuous-time uncertain systems with signal-transmission delay, quantized measurements and packet dropouts has been given in [23]. For $H_\infty$ filtering of linear systems with packet dropouts (see, e.g., [24–26]). Moreover, the problem of $H_\infty$ filtering for discrete-time T-S fuzzy systems with measurement quantization and packet dropouts has been studied in [27]. Similar problem for linear systems has been considered in [28]. We have also been concerned about the study of feedback control problem and delay problem for networked control systems, see [29] and [30].

This paper focuses on the relaxed problem of designing $H_\infty$ filter for uncertain discrete-time systems with quantized measurements and packet dropouts. Via introducing auxiliary relaxed variables, less conservatism results are obtained in terms of LMIs, which can not only mitigate the effects of quantization and packet dropouts but also ensure a prescribed $H_\infty$ performance and stochastic stability. Finally, we will illustrate the effectiveness and the advantage of our main results by two examples. The contributions of this paper are: (i) The problem of robust $H_\infty$ filtering for discrete-time systems with polytopic uncertainties subject to quantization and packet dropouts has been investigated. (ii) Compared with the existing literatures, the main advantage of the proposed method is the reduced conservativeness.

**Notations.** The symbol $*$ induces a symmetric structure in LMIs. Generally, for a square matrix $A$, $A^T$ denote its transpose and $He\{A\}$ denotes $(A + A^T)$. In addition, $\mathbb{E}\{x\}$ denotes expectation of $x$ and $Pr\{\cdot\}$ stands for the occurrence of an event. Matrices are assumed to have compatible dimensions.

## 2. Problem statement and preliminaries

The filtering system we consider here consists four parts: a plant modeled by discrete-time systems with polytopic uncertainties, a logarithmic quantizer, a full-order filter and unreliable communication channel between them. The quantized measurement is transmitted to the filter across an unreliable communication channel where packet dropouts may be occurred.

Consider the following discrete-time system with polytopic uncertainties described by state-space equations:

$$\begin{aligned} x(k+1) &= A(\eta)x(k) + B(\eta)w(k) \\ y(k) &= C(\eta)x(k) + D(\eta)w(k) \\ y_q(k) &= Q(y(k)) \end{aligned} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ is the state variable, $y(k) \in \mathcal{R}^f$ is the measurement and $w(k) \in \mathcal{R}^m$ is the noise signal that is assumed to be the arbitrary signal in $l_2[0, \infty)$.

**Remark 1.** The system can also be used to deal with the case when the process noise and the measurement noise are different. Assume that the process noise is $w(k)$ and the measurement noise is $v(k)$, we can easily define $\bar{w}(k) = [w^T(k), v^T(k)]^T, B_1(\eta) = [B(\eta), 0], D_1(\eta) = [0, D(\eta)]$ in system (1).

The matrices $A(\eta)$, $B(\eta)$, $C(\eta)$ and $D(\eta)$ belong to the following polyhedron:

$$\begin{aligned} \Omega = \{&[A(\eta), B(\eta), C(\eta), D(\eta)] = \sum_{i=1}^{r} \eta_i [A_i, B_i, C_i, D_i], \\ &\sum_{i=1}^{r} \eta_i = 1, \eta_i \geq 0\}. \end{aligned} \tag{2}$$

$Q(y(k))$ is the quantized measurement. Here $Q(\cdot) = [Q_1(\cdot) Q_2(\cdot) \cdots Q_f(\cdot)]^T$ is a static time-invariant logarithmic quantizer given by Fu and Xie [13]:

$$Q_j(y) = \begin{cases} v_i^{(j)} & 0 \leq (1/(1+\delta_j))v_i^{(j)} < y \leq (1/(1-\delta_j))v_i^{(j)} \\ 0 & y = 0 \\ -Q_j(-y) & y < 0 \end{cases} \tag{3}$$

$$\delta_j = \frac{1-\rho_j}{1+\rho_j}, 0 < \rho_j < 1, v_i^{(j)} > 0. \tag{4}$$