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Mean square H_∞ synchronization of coupled stochastic partial differential systems

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ABSTRACT

In this paper, the criterion and control are considered for the mean square H_∞ synchronization of coupled stochastic partial differential systems (SPDSs). Based on the integral Lyapunov-like functional and by virtue of completing squares technique, a sufficient criterion is provided to guarantee the mean square H_∞ synchronization. The effect of spatial domain on the mean square H_∞ synchronization is also embodied in this criterion. When the coupled SPDSs cannot achieve the mean square H_∞ synchronization, the adaptive controllers are adopted for the coupled SPDSs, and the effectiveness of the adaptive controllers is verified via a rigorous mathematical analysis. When the number of the nodes in the complex networks is large, pinning control is a natural choice. The adaptive pinning control strategy is also presented and a criterion is obtained which guarantees the mean square H_∞ synchronization of coupled SPDSs. Numerical experiments are also given to illustrate the correctness of our results.

1. Introduction

In the last two decades, with the fast development of the internet technology, great conveniences were brought by the networks, and the complex networks have been one of the research focuses. The topology and the dynamical properties of the networks have been widely studied, see [1] and the references therein. Synchronization, as one of the significant properties of the complex networks, has attracted a lots of attentions and a great many of results on the synchronization have been published, see [2–7].

As well known, many phenomena, which can be found in chemical engineering, biology, population dynamics, neurophysiology and biodynamics, etc., have been modeled in the spatio-temporal domain by partial differential systems (PDSs) [8–10]. The states of these systems depend on not only the time but also the space. The synchronization of coupled PDSs have also been paid a great deal of concern, see [11–13]. Moreover, external disturbances are ubiquitous in the real world and some of them can be modeled by white noises. These systems with white noises can be described by stochastic differential systems driven by Brownian motion [14–17]. The synchronization of coupled stochastic differential systems has also been well studied. In [18], the authors provided the sufficient conditions to guarantee the mean square asymptotical synchronization of stochastic neural networks with time-delays. The Lyapunov functional and linear matrix inequalities (LMIs) approaches were used and control gain was obtained in that paper. In [19] the authors considered the adaptive synchronization of stochastic neural networks of

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neutral-type with mixed time-delays, using the LaShall-type invariance principle for the stochastic delay differential equations, they obtained the LMIs criterion with adaptive synchronization controllers. Moreover, the hybrid and impulsive control for the stochastic complex networks with nonidentical nodes was investigated in [20].

There do exist disturbances which cannot be modeled by white noises and they enter the systems as additive noises. External disturbances can lead a given system to an unanticipated state and even destroy the synchronization. Hence, it is necessary to study the disturbance resisting ability, and that is the H_∞ synchronization. The robust H_∞ synchronization was first brought out in [21,22], where the authors proposed a method to deal with the robust nonlinear H_∞ master-slave synchronization for chaotic Lur'e systems with applications to secure communication. However, a mathematical definition of H_∞ synchronization was not presented in those paper and the methods used in those paper were not generalized to other systems. Until 2007, Hou et al. [23] provided the mathematical definition of the H_∞ synchronization and the Lyapunov function method used in that paper was regarded as a classical method to study the problem of H_∞ synchronization. After that work, a good many results on the H_∞ synchronization were reported and various dynamical systems have been studied, see [24–31] and the references therein. However, to our knowledge, there are few paper on the H_∞ synchronization of the disturbed coupled stochastic partial differential systems. Moreover, with the increasing interests of the synchronization control, adaptive synchronization control has become a hot topic, see [12,32,33]. When the number of the subsystems of the complex networks is large, pinning control is a natural choice [34–36].

Motivated by the above discussion, in this paper, we consider the mean square H_∞ synchronization for disturbed coupled stochastic partial differential systems (SPDSs). Using the integral-form Lyapunov functional, stochastic analysis and completing squares technique, we obtain the sufficient conditions to guarantee the mean square H_∞ synchronization. The adaptive synchronization control for the SPDSs is also considered and the criterion of the mean square H_∞ synchronization with adaptive controller is presented. Pining control is also investigated in brief. At last, numerical examples are given to illustrated our results obtained in this paper.

2. Preliminaries and model description

The following N-coupled stochastic partial differential systems with disturbances are considered on a given undirected connected graph $\mathcal{G}$

$$dy_i(x,t) = \left[f(y_i(x,t)) + B\nabla^2 y_i(x,t) + c\sum_{j=1}^{N} g_{ij}y_j(x,t) + v_i(x,t) \right]dt + g(y_i(x,t))dB(t), x \in U, i = 1, 2, \ldots, N, \tag{2.1}$$

where $f(\cdot)$ is a smooth nonlinear function, $y_i = (y_{i1}, y_{i2}, \ldots, y_{in})^T$ is the state variable of the ith node. $x = (x_1, x_2) \in U \subset \mathbb{R}^2$ and $t \geq 0$ are the spatial and time variables and $|x_i| \leq l_i$, $i = 1, 2$. The positive constant c stands for the coupling strength. $G = (g_{ij})_{N\times N}$ is the coupling configuration matrix and g_{ij} is defined as follows: if there exists information changes between node i and node j, then $g_{ij} = g_{ji} = 1$, otherwise $g_{ij} = g_{ji} = 0$, $i \neq j$. Here we assume that G is a diffusive matrix, that is, $\sum_{j=1}^{N} g_{ij} = 0$ for any $i = 1, 2, \ldots, N$. We also assume that the constant matrix B is symmetric and positive. $\nabla^2 y_i(x, t)$ is defined as

$$\nabla^2 y_i(x,t) = \frac{\partial^2 y_i(x,t)}{\partial x_1^2} + \frac{\partial^2 y_i(x,t)}{\partial x_2^2}.$$

$B(t)$ is an 1-dimensional Brownian motion in a complete probability space $\{\Omega, \mathcal{F}, P\}$.

The boundary conditions and initial values are posed as follows:

$$y_i(x,t) = 0, \quad x \in \partial U, \quad y_i(x,0) = y_i(x). \tag{2.2}$$

where ∂U stands for the boundary of the spatial domain U.

Let $s(x, t)$ be the function which $y_i(x, t)$ to be synchronized and satisfies

$$ds(x,t) = [f(s(x,t)) + B\nabla^2 s(x,t)]dt + g(s(x,t))dB(t). \tag{2.3}$$

The boundary conditions and the initial values are posed as

$$s(x,t) = 0, \quad x \in \partial U, \quad s(x,0) = \varphi(x). \tag{2.4}$$

As a standing assumption, we suppose that there exists a unique solution for systems (2.1) and (2.2) and systems (2.3) and (2.4), respectively.

Take $e_i(x,t) = y_i(x,t) - s(x,t)$ be the synchronization error. Using the fact $\sum_{j=1}^{N} g_{ij} = 0$ for any $i = 1, 2, \ldots, N$, we have the following error dynamical system

$$de_i = \left[f(y_i) - f(s) + B\nabla^2 e_i + c\sum_{j=1}^{N} g_{ij}e_j + v_i \right]dt + [g(y_i) - g(s)]dB(t), \quad i = 1, 2, \ldots, N. \tag{2.5}$$

Here and in the sequel, the variables (x, t) are compressed for the convenience. Obviously, $e_i(x,t) = 0$ when $x \in \partial U$.

Now we provide some lemma, definition and assumption to end this section. All of them are useful in the sequel analysis.