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The Zagreb indices of four operations on graphs

Hanyuan Deng [a], D. Sarala [b], S.K. Ayyaswamy [b], S. Balachandran [b,*]

[a] College of Mathematics and Computer Science, Hunan Normal University, Changsha, Hunan 410081, PR China
[b] Department of Mathematics, School of Humanities and Sciences, SASTRA University, Thanjavur, India

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ABSTRACT

For a (molecular) graph, the first Zagreb index M_1 is equal to the sum of squares of the degrees of vertices, and the second Zagreb index M_2 is equal to the sum of the products of the degrees of pairs of adjacent vertices. In this work, we study the first and second Zagreb indices of four operations on graphs.

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1. Introduction

Throughout this paper we consider only simple connected graphs, i.e., connected graphs without loops and multiple edges. For a graph $G = (V, E)$ with vertex set $V = V(G)$ and edge set $E = E(G)$, the degree of a vertex v in G is the number of edges incident to v and denoted by $d_G(v)$.

A graphical invariant is a number related to a graph which is structurally invariant. In chemical graph theory, these invariant numbers are also known as the topological indices. One of the oldest graph invariants is the well-known Zagreb index first introduced more than forty years ago by Gutman and Trinajestić [13], where Gutman and Trinajstć examined the dependence of total π-electron energy on molecular structure, and this was elaborated on in [12]. For a (molecular) graph G, the first Zagreb index $M_1(G)$ and the second Zagreb index $M_2(G)$ are, respectively, defined as follows:

$$M_1 = M_1(G) = \sum_{v\in V(G)} d_G^2(v), \quad M_2 = M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

Also, $M_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]$. For more details on these indices see the recent papers [1,2,4,5,10,14,20,25–29] and the references therein. The zeroth-order general Randić index is a more general case of the first Zagreb index [15,16] and see survey paper on Randić index [18].

The extremal graphs that maximize or minimize the Zagreb indices within certain classes of trees have been studied intensively in recent years [7,19,21–23,26,30]. Recently, Goubko and Gutman [9,11] characterized the trees with the minimum first and second Zagreb indices among the trees with fixed number of pendent vertices. After that, Lin [24] characterized the trees with fixed number of vertices of degree two that maximize and minimize the First Zagreb index. In [17], exact expressions for the first and second Zagreb index of graph operations containing the Cartesian product, composition, join, disjunction, and symmetric difference of graphs were presented.

In this work, we will study the first and second Zagreb indices of four operations on graphs. For this purpose, we recall some operations on graphs in the following.

* Corresponding author. Tel.: +91 9976964951.
E-mail address: bala@maths.sastra.edu (S. Balachandran).

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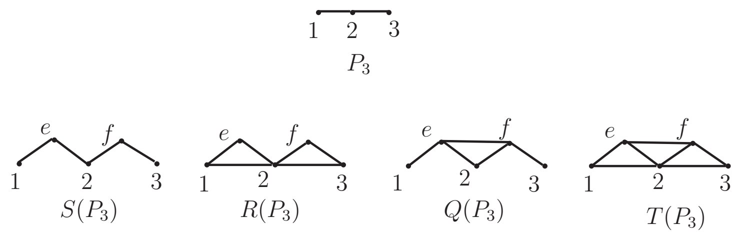

Fig. 1. P_3, $S(P_3)$, $R(P_3)$, $Q(P_3)$ *and* $T(P_3)$.

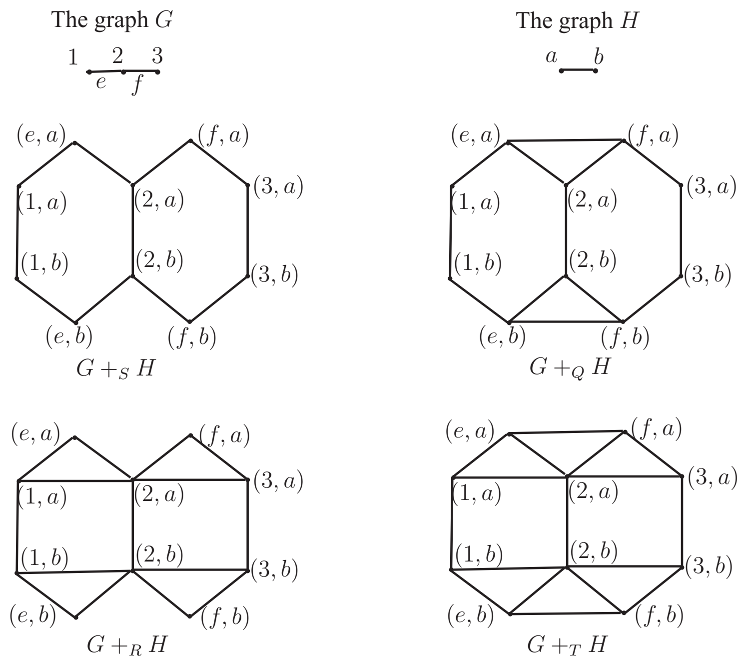

Fig. 2. Graphs G and H and $G +_F H$.

The Cartesian product of two connected graphs G_1 and G_2, which is denoted by $G_1 \times G_2$, is a graph such that the set of vertices is $V(G_1) \times V(G_2)$ and two vertices $u = (u_1, v_1)$ and $v = (u_2, v_2)$ of $G_1 \times G_2$ are adjacent if and only if $[u_1 = u_2$ and v_1 is adjacent with v_2 in $G_2]$ or $[v_1 = v_2$ and u_1 is adjacent with u_2 in $G_1]$.

For a connected graph G, there are four related graphs as follows:

(a) $S(G)$ is the graph obtained by inserting an additional vertex in each edge of G. Equivalently, each edge of G is replaced by a path of length 2.
(b) $R(G)$ is obtained from G by adding a new vertex corresponding to each edge of G, then joining each new vertex to the end vertices of the corresponding edge.
(c) $Q(G)$ is obtained from G by inserting a new vertex into each edge of G, then joining with edges those pairs of new vertices on adjacent edges of G.
(d) $T(G)$ has as its vertices the edges and vertices of G. Adjacency in $T(G)$ is defined as adjacency or incidence for the corresponding elements of G.

The graphs $S(G)$ and $T(G)$ are called the subdivision and total graph of G, respectively. For more details on these operations we refer the reader to [3].

If G is P_3, then $S(P_3)$, $R(P_3)$, $Q(P_3)$ and $T(P_3)$ are shown in Fig. 1.

Suppose that G_1 and G_2 are two connected graphs. Based on these operations above, Eliasi and Taeri [6] introduced four new operations on these graphs in the following:

Let $F \in \{S, R, Q, T\}$. The F-sum of G_1 and G_2, denoted by $G_1 +_F G_2$, is defined by $F(G_1) \times G_2 - E^*$, where $E^* = \{(u, v_1)(u, v_2) \in E(F(G_1) \times G_2) : u \in V(F(G_1)) - V(G_1), v_1v_2 \in E(G_2)\}$, i.e., $G_1 +_F G_2$ is a graph with the set of vertices $V(G_1 +_F G_2) = (V(G_1) \cup E(G_1)) \times V(G_2)$ and two vertices (u_1, u_2) and (v_1, v_2) of $G_1 +_F G_2$ are adjacent if and only if $[u_1 = v_1 \in V(G_1)$ and $u_2v_2 \in E(G_2)]$ or $[u_2 = v_2 \in V(G_2)$ and $u_1v_1 \in E(F(G_1))]$.

$P_3 +_S P_2$, $P_3 +_R P_2$, $P_3 +_Q P_2$ and $P_3 +_T P_2$ are shown in Fig. 2.

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